

0) Background.
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Let X be a non-empty subset of a finite directed graph G . A vertex of X is called a *head* of X in G if it is a predecessor of all the other vertices of X . We denote by $H_G(X)$ the set of the heads of X in G . X is called *headed* if $H(X) \neq \emptyset$ and *totally headed* if all the non-empty subsets of X are headed.

Given a function $f: S \rightarrow G$, where S is a topological space, we denote by capital letter V the set of all the f -counterimages of $v \in G$, and, if we want to emphasize the function f , we write $V^f = f^{-1}(v)$.

We call *image-envelope* of a point $x \in S$ by f , and we denote by $\langle f(x) \rangle$, the set of vertices, such that the closure of their f -counterimages include the point i.e. $v \in \langle f(x) \rangle \Leftrightarrow x \in \bar{V}^f$.

A function $f: S \rightarrow G$ is called *o-regular*, if, for all different $v, w \in G$, such that v is not a predecessor of w , it is $V \cap \bar{W} = \emptyset$. We proved that f is o-regular iff:

- i) $\langle f(x) \rangle$ is headed, $\forall x \in S$;
- ii) $f(x) \in H(\langle f(x) \rangle)$, $\forall x \in S$. (See [5], Proposition 2).

So it is natural to define a more restrictive class of functions by saying that a function $f: S \rightarrow G$ is *completely o-regular* (or simply *c.o-regular*) if

- i') $\langle f(x) \rangle$ is totally headed, $\forall x \in S$;
- ii') $f(x) \in H(\langle f(x) \rangle)$, $\forall x \in S$.

Afterwards we also consider functions satisfying only condition i', which we call *completely quasi regular* functions. In [5] we proved that a completely quasi regular function can be replaced by a c.o-regular one by constructing the o-patterns of the function (where an *o-pattern* of a function $f: S \rightarrow G$ is a function $g: S \rightarrow G$ such that $g(x) \in H(\langle f(x) \rangle)$, $\forall x \in S$). In the case of pairs of topological spaces S, S' and of pairs of graphs G, G' in [5] in order to introduce the o-patterns, we gave the definition of *balanced* function i.e. of a function $f: S, S' \rightarrow G, G'$ such that $\langle f(x') \rangle = \langle f'(x') \rangle$, $\forall x' \in S'$. With reference to this we remember that if the subspace S' is open in S , all the functions are balanced.

II) Enlargability of sets in a uniform space.
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DEFINITION 1. - Let (S, \mathcal{W}) be a uniform space, where the filter \mathcal{W} is the uniformity of S . Given a vicinity $W \in \mathcal{W}$, we put $W(x) = \{y \in S / (x, y) \in W\}$, $\forall x \in S$, and $W(X) = \bigcup_{x \in X} W(x)$, $\forall X \subset S$.

' REMARK. - If (S, d) is a metric space the subsets $W^\epsilon = \{(n, a) \in S \times S /$