tion results in income dispersion affecting also the firms' price and quantity choices, through changes in the equilibrium mark-up. Concluding remarks are gathered in Section 5.

2 Market demand and income dispersion

We consider a population of consumers who differ only in their income I. The latter is distributed according to a continuous, differentiable, unimodal density $f(I, \theta)$, defined over the positive interval $[I_{\min}, I_{\max}]$. In order to focus on the effects of income inequality, in the sequel we interpret the parameter $\theta \in \Theta$ as a mean preserving spread, so that an increase in θ can be seen as an increase in income dispersion which leaves average income unchanged.

Consumers' preferences are identical and represented by the following utility function:

$$U = U\left(x_0, \left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right) \qquad (1)$$

where x_0 is a numéraire homogeneous commodity and x_i , $i = 1, ..., n$, are the different varieties of a CES composite differentiated good $y = \left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, where $\sigma > 1$ is the constant elasticity of substitution across varieties. We depart from the standard specifications of this Dixit-Stiglitz framework, by assuming that (1) is non-homothetic, in order to generate Engel's curves which are not unit-elastic in income. Clearly, the strict proportionality between demand and income associated to homothetic preferences would not leave any role to income distribution in the analysis of demand, the only relevant parameter being the income mean (aggregate) value.

Each consumer maximizes (1), given the linear budget constraint

$$x_0 + \sum_{i=1}^{n} p_i x_i = I$$

Through a two-stage budgeting procedure, the solution of this maximization problem yields the following demand function for each variety x_i:

$$x_i^d = \left(\frac{p_i}{q}\right)^{-\sigma} \frac{s}{q} \qquad (2)$$

where s is the consumer's expenditure in the differentiated good and q is the (dual) price index defined as

$$q = \left(\sum_{i=1}^{n} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

By substituting (2) into (1) for all x_i and recalling that $x_0 = I - s$, it is possible to determine the optimal value of s as a function of I and q, $s = s(I, q)$, and therefore the marshallian demand for the differentiated good and the numeraire.

$$\begin{aligned} y &= \frac{s(I,q)}{q} \\ x_0 &= I - s(I,q) \end{aligned}$$

The marshallian demand for variety x_i is therefore

$$x_i^d = \left(\frac{p_i}{q} \right)^{-\sigma} \frac{s(I,q)}{q}$$

By aggregating over consumers we obtain the market demand for variety i:

$$X_i^d = \left(\frac{p_i}{q} \right)^{-\sigma} \frac{1}{q} S(q, \theta) \tag{3}$$

where

$$S(q, \theta) = \int_{I_{\min}}^{I_{\max}} s(I, q) f(I, \theta) \, dI$$

Given the heterogeneity of consumers with respect to income, market demand is in principle affected by the parameters of income distribution. However, with homethetic preferences the s function (and the demand function) would be linear in income and the mean preserving spread parameter would not affect aggregate expenditure S. Our non-homotheticity hypothesis allows for a concave or convex shape of s, so that θ actually influences S and market demand X_i^d. In particular, the following proposition holds:

Proposition 1 *If the differentiated good is a necessary good, $\partial X_i^d/\partial\theta < 0$, i.e. an increase in income dispersion decreases market demand. If the differentiated good is a luxury good, $\partial X_i^d/\partial\theta > 0$, i.e. an increase in income dispersion raises market demand.*

The proof is omitted, as it is a direct application of the general result that the expected value of a concave (convex) function is decreasing (increasing) in any mean preserving spread parameter (Hirshleifer and Riley, 1992, p. 112).

This result is rather intuitive. An increase in income dispersion implies an increase in the density of low income and high income consumers, with a shrinking of the middle class. The Engel curve of a necessary good is concave and therefore the increase in demand from the newly rich consumers does not compensate the decrease in demand by the newly poor consumers. The opposite applies in the case of a luxury good. A simple example where aggregate expenditure is a linear function of the dispersion parameter is provided below.

Example. Consider the non-homothetic utility functions (Chou and Talmain, 1996)

$$U_1 = \sqrt{x_0} + \ln\left(\left(\sum x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right)$$
$$U_2 = -\frac{1}{x_0} + \ln\left(\left(\sum x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right)$$

to which there correspond the individual marshallian demand functions for variety i:

$$x_{i1}^d = \left(\frac{p_i}{q}\right)^{-\sigma}\frac{2}{q}\left(\sqrt{1+I}-1\right)$$
$$x_{i2}^d = \left(\frac{p_i}{q}\right)^{-\sigma}\frac{1}{q}\left(I+\frac{1}{2}-\frac{1}{2}\sqrt{(4I+1)}\right)$$

where x_{i1}^d is concave and x_{i2}^d is convex in income. Assume now that income is distributed according to the density

$$f(I,\theta) = \theta + 6(1-\theta)I(1-I)$$

defined over the support $[0,1]$.[2] By aggregating the individual demand curves, we obtain market demand functions linear in θ of the type:

$$X_{i1}^d = \left(\frac{p_i}{q}\right)^{-\sigma} \frac{1}{q} (\alpha_1 - \beta_1 \theta)$$
$$X_{i2}^d = \left(\frac{p_i}{q}\right)^{-\sigma} \frac{1}{q} (\alpha_2 + \beta_2 \theta)$$

where α_j and β_j $(j = 1,2)$ are positive numbers.[3] As expected, X_{i1}^d is decreasing (X_{i2}^d is increasing) in θ.

We now apply the above demand framework in the analysis of market equilibrium.

3 Pricing and market equilibrium: the case with exogenous mark-up

Following the standard Dixit-Stiglitz approach, we assume that each firm faces the following cost function

$$C(x_i) = a + cx_i$$

If each firm maximizes its own profits under the demand constraint (3) and taking q as given, the symmetric short run equilibrium price is

$$p_i = p = c\frac{\sigma}{\sigma - 1} \tag{4}$$

and since $q = pn^{\frac{1}{1-\sigma}}$,

$$X_i = X = \frac{S\left(pn^{\frac{1}{1-\sigma}}, \theta\right)}{pn} \tag{5}$$

According to equation (4), the equilibrium mark-up is fully determinated by the exogenous cost and demand parameters, and is therefore independent

[2] This density, a mixture of a uniform and a quadratic beta distribution, is unimodal and symmetric. It is easy to check that the parameter $\theta \in [0,1]$ is a mean preserving spread, so that an increase in θ increases income dispersion.

[3] In particular we have $\alpha_1 \simeq 0.4426$, $\beta_1 \simeq 0.0047$, $\alpha_2 \simeq 0.1443$, $\beta_2 \simeq 0.0074$.