the whole range of p – the sign of η_θ in area D is clearly still ambigous. Sorting this out would enable us to determine the behaviour of η over the whole range of p. The properties of the income distribution which deliver uniqueness of $\widehat{p}$ are discussed in the next section.

3 Income share elasticity and the price elasticity of demand

Ideally, one would expect to pin down a unique value $\widehat{p}$, such that $\eta_\theta > 0$ for all $p < \widehat{p}$ and $\eta_\theta < 0$ for all $p > \widehat{p}$. Given that $\eta_\theta > 0$ for $p \in A$ and $\eta_\theta < 0$ for $p \in \widehat{B}$, η_θ crosses zero from above at the left boundary of $\widehat{B}$, i.e.at $\widehat{p}$. In order to define the conditions for $\widehat{p}$ to be unique, we first notice that the derivative of η_θ with respect to p is

$$\begin{aligned} \eta_{\theta p} &= \frac{\eta(p,\theta)}{f(p,\theta)}\left(f_{p\theta}(p,\theta) - \frac{f_p(p,\theta)}{f(p,\theta)} f_\theta(p,\theta)\right) \\ &\quad + \left(\eta_p + \frac{\eta^2}{p}\right)\left[\frac{f_\theta(p,\theta)}{f(p,\theta)} + \frac{F_\theta(p,\theta)}{1-F(p,\theta)}\right] \end{aligned} \tag{6}$$

which, for $\eta_\theta = 0$ collapses to

$$\eta_{\theta p|\eta_\theta=0} = \frac{\eta(p,\theta)}{p}\Pi_\theta\left(p,\theta\right) \tag{7}$$

where $\Pi_\theta\left(y,\theta\right)$ is the derivative with respect to θ of the income share elasticity (Esteban, 1986). The latter is defined as

$$\Pi\left(y,\theta\right) = 1 + \frac{y f_y\left(y,\theta\right)}{f\left(y,\theta\right)}$$

and measures the percentage change of the income share accruing to individuals of income y, given a marginal change in y.[7] Esteban shows that there is a one-to-one correspondence between $f\left(y,\cdot\right)$ and $\Pi\left(y,\cdot\right)$, so that any given distribution can be characterized in terms of Π.

Therefore, given (7),

$$\text{at } \eta_\theta = 0, \quad sign\left[\eta_{\theta p}(p,\theta)\right] = sign\left[\Pi_\theta(p,\theta)\right] \tag{8}$$

This is particularly convenient, as the Π function typically exhibits some useful regularity properties.

[7] Formally, $\Pi = \lim_{h\to 0}\frac{1}{\mu}\int_y^{y+h} x f\left(x,\theta\right)dx$, where μ is the mean income (Esteban 1986, p.441).

We are now in the position to establish the following general proposition.

Proposition 2 If the distribution $f(y,\theta)$ and the corresponding income share elasticity $\Pi(y,\theta)$ are such that (a) $\Pi_\theta(y,\theta)$ is monotonically increasing in y and crosses zero, and (b) $\lim_{p\to y_M}\eta_\theta < 0$, then there exists one value $\widehat{p}$ such that $\eta_\theta(p,\theta) > 0$ for $p < \widehat{p}$ and $\eta_\theta(p,\theta) < 0$ for $p > \widehat{p}$.

Proof By Proposition 1 there exists a $\widehat{p}$ which is the lowest p such that $\eta_\theta(p,\theta)$ crosses zero, obviously from above. Condition (a) together with (8) imply that $\Pi_\theta(\widetilde{p},\theta) = 0$ at some unique $\widetilde{p} > \widehat{p}$. This implies that $\widehat{p}$ is the unique value of p at which η_θ is zero. To see this, notice that by condition (b), if additional such points existed, they should be even in number. Suppose they are two (the proof applies trivially for any even number), and call them $\widehat{p}_1$ and $\widehat{p}_2$, $\widehat{p}_1 < \widehat{p}_2$. Obviously, $\eta_{\theta p}$ will be positive at $\widehat{p}_1$ and negative at $\widehat{p}_2$. Two possibilities arise: (*i*) $\widehat{p}_1$ and $\widehat{p}_2$ are both lower or higher than $\widetilde{p}$; (*ii*) $\widehat{p}_1 < \widetilde{p} < \widehat{p}_2$. Case (*i*) is ruled out by (8); case (*ii*) is ruled out by (8) together with condition (a). □

It should be noticed that conditions (a) and (b) of Proposition 2 are verified for many widely used distributions, such as those quoted in f.note 2.

One implication of Proposition 2 is that the interval $\widehat{B}$ identified by Proposition 1 is unambiguously defined as $(\widehat{p}, y_B)$. Figure 2 brings this out by showing a possible behaviour of the sensitivity of the elasticity of market demand to θ, for different values of p.

Figure 2 about here

Why is it that, for prices lying between $\widehat{p}$ and y_B, an increase in income concentration generates both an increase in demand and an increase in its elasticity (which for constant marginal costs would imply non competitive firms setting a lower price)? Prices in that interval are prices at which the higher income individuals getting poorer are still able to buy, while lower income individuals getting richer are eventually allowed to enter the market. The additional demand accruing at these prices is therefore due to the latter - those who descend into the middle class from the upper tail of the distribution were already buyers, and keep buying after the distributional change. However, this overall movement from the tails towards the central area of the distribution is such that for prices belonging to $\widehat{B}$, there are more consumers actually buying, whose reservation price is close to the set price. This implies, for example, that non competitive firms perceive a weaker incentive to exploit an intensive margin on higher income consumers, and a stronger incentive to acquire new consumers at the margin by keeping lower prices.[8] Demand increases and becomes more elastic simply because there are indeed new consumers entering the market, but also more consumers whose decision to enter or exit the market is now very sensible to small variations in prices. Notice that these observations are consistent with the fact that a positive comovement of demand and demand elasticity is observed only in $\widehat{B}$, i.e., it is peculiar of an intermediate portion of the demand curve, as defined by $\widehat{B}$. Moreover, they apply to whatever unimodal distribution, once concentration towards central income values is considered, and this explains the generality of our result. As a notable example, in the next section we apply the results of Propositions 1 and 2 to the lognormal distribution.

4 An example: income dispersion with lognormal distribution

Assume that income is distributed lognormally. This is a particularly remarkable case, since – as is well known – the lognormal distribution is perhaps the model most frequently used to describe actual income frequencies.[9]

We standardize mean income equal to unity, so that the density and distribution functions take the form[10]

$$f(y,\theta) = \frac{1}{y\sqrt{2\pi \ln\theta}} \exp\left(-\frac{\left(\ln y + \frac{1}{2}\ln\theta\right)^2}{2\ln\theta} \right)$$

$$F(y,\theta) = \int_0^y f(x,\theta)dx = \frac{1}{2}\left[1 + \Phi\left(\frac{1}{4}\sqrt{2}\frac{2\ln y + \ln\theta}{\ln^{\left(\frac{1}{2}\right)}\theta} \right) \right]$$

[8] This may offer a general explanation for the empirical evidence discussed by Frankel and Gould (2001), who find a causal link running from income distribution in urban areas to retail prices: according to their estimates, greater inequality is indeed associated with an increase in retail prices paid by lower middle-class consumers.

[9] It is well known that the lognormal distribution fits satisfactorily the actual income distribution for central income values, while it is unsatisfactory in the tails, i.e. for extreme income values (for an evaluation of the empirical performance of various distributions, see e.g. Majumder and Chakravarty, 1990). Since the phenomenon we are interested in is peculiar of intermediate intervals, the lognormality assumption seems worth investigating. We recall that, if reservation prices are proportional to incomes, they also are lognormally distributed.

[10] Given a generic lognormal distribution $f(y,\theta) = (y\sqrt{2\pi\ln\theta})^{-1}\exp\left(-\frac{(\ln y-\zeta)^2}{2\ln\theta}\right)$, the mean is $\mu = e^{\zeta}\sqrt{\theta}$. Clearly, by imposing $\mu = 1$ one constrains the parameters θ and ζ according to the restriction $\varsigma = -\frac{1}{2}\ln\theta$. Note, in particular, that income variance is $\sigma^2 = \theta - 1 > 0$.