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DUALS OF INDUCTIVE AND PROJECTIVE LIMITS OF MOSCATELLI TYPE

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Abstract. *In this note we shall provide duality results between the general inductive and projective limits of Moscatelli type. These extend the corresponding duality results in the case of Fréchet and LB-spaces due to J. Bonet and S. Dierolf.*

1. PRELIMINARIES AND DEFINITIONS

From now on, $(L, \|\,\|)$ will denote a normal Banach sequence space i.e. a Banach sequence space satisfying:

(α) $\varphi \subset L \subset \omega$ algebraically and the inclusion $(L, \|\,\|) \to \omega$ is continuous (here ω and φ stand for $\mathbb{K}^{\mathbb{N}} = \prod_{k\in\mathbb{N}} \mathbb{K}$ and $\bigoplus_{k\in\mathbb{N}} \mathbb{K}$ respectively).

(β) $\forall a = (a_k) \in L, \forall b = (b_k) \in \omega$ with $|b_k| \leq |a_k|$ $(k \in N)$, we have $b \in L$ and $\|b\| \leq \|a\|$.

Clearly every projection onto the first n coordinates $p_n : \omega \to \omega, (a_k)_{k\in\mathbb{N}} \to ((a_k)_{k\leq n}, (0)_{k>n})$ induces a norm-decreasing endomorphism on L. Other properties on $(L, \|\,\|)$ we may require are the following:

(γ) $\|a\| = \lim \|p_n(a)\|$ $(n \to \infty), \forall a \in L$.

(ε) $\lim \|a - p_n(a)\| = 0$ $(n \to \infty), \forall a \in L$ (i.e. φ is dense in $(L, \|\,\|)$.

(δ) If $a \in \omega$, and $\sup_{n\in\mathbb{N}} \|p_n(a)\| < \infty$, then $a \in L$ and $\|a\| = \lim \|p_n(a)\|$ $(n \to \infty)$.

Unexplained terminology as in [6, 7, 10].

Let $(L, \|\,\|)$ be a normal Banach sequence space, let Y and X be locally convex spaces and $f : Y \to X$ a continuous linear mapping. For every $n \in \mathbb{N}$, we define $F_n := \prod_{k<n} Y \times L((X)_{k\geq n})$ provided with the topology of such a finite topological product. Note that this topology is generated by the seminorms: $(x_k)_{k\in\mathbb{N}} \in F_n \to \|((q(x_k))_{k<n}, (p(x_k))_{k\geq n})\|$ with $q \in cs(Y)$, $p \in cs(X)$.

For every $n \in \mathbb{N}$, we also define the mapping: $g_n : F_{n+1} \to F_n$, $(x_k)_{k\in\mathbb{N}} \to ((x_k)_{k<n}, f(x_n), (x_k)_{k>n})$. Clearly g_n $(n \in \mathbb{N})$ is a continuous linear mapping and F_n $(n \in \mathbb{N})$ is Hausdorff (resp. complete, metrizable, normable) if and only if X and Y have the same property. Moreover the spaces X and Y are complemented subspaces of F_n for $n \geq 1$ and $n \geq 2$ respectively.

We define the *projective limit F of Moscatelli type w.r.t. (with respect to)* $(L, \|\,\|), Y, X$ and $f : Y \to X$ by $F = \operatorname{proj}_{n\in\mathbb{N}} (F_n, g_n)$

Observe that F is Hausdorff, (resp. complete, metrizable) if and only if Y and X have the same property.

We shall first present another description of the space F similar to the one given in [3] for the case of Banach spaces X and Y.

1.1 Proposition. *Let $(L, \|\,\|)$ be a normal Banach sequence space, let Y and X be locally convex spaces and $f : Y \to X$ a continous linear mapping. The corresponding projective limit of Moscatelli type F coincides algebraically with $\{y = (y_k)_{k\in\mathbb{N}} \in \prod_{k\in\mathbb{N}} Y : (f(y_k))_{k\in\mathbb{N}} \in L(X)\}$ and F has the initial topology w.r.t. the inclusion $j : F \to \prod_{k\in\mathbb{N}} Y$ and the linear mapping $\tilde{f} : F \to L(X), (y_k)_{k\in\mathbb{N}} \to (f(y_k))_{k\in\mathbb{N}}$.*

Proof. Denote by H the space $\{(y_k)_{k\in\mathbb{N}} \in \prod_{k\in\mathbb{N}} Y : (f(y_k))_{k\in\mathbb{N}} \in L(X)\}$ carrying the initial topology w.r.t. j and $\tilde{f}$ and define $\gamma : F \to H, (z^k)_{k\in\mathbb{N}} \to \left(z_k^{k+1}\right)_{k\in\mathbb{N}}$. It is easy to show that γ is linear, bijective, continuous and open. ∎

Note that we obtain the same space F if we take the closure of $f(Y)$ in X instead of X. Therefore we may assume without loss of generality that f has dense range.

2. BOUNDED SETS

Before dealing with the duality between inductive and projective limits of Moscatelli type we need some preparation. Our first two results are essentially well-known (see[10] for $L = l_1$ or [12])

If Z is a locally convex space, $\mathfrak{b}(Z)$ will denote the family of all the bounded sets in Z and we shall always consider them closed and absolutely convex. Besides $Z'_b := [Z, \beta(Z', Z)]$ will stand for the strong dual of Z.

2.1 Lemma. *Let $(L, \|\,\|)$ be a normal Banach sequence space and let Z be a metrizable space. Then for every $\mathfrak{B} \in \mathfrak{b}(L(Z))$, there exist $B \in \mathfrak{b}(Z)$ such that $\mathfrak{B} \in \mathfrak{b}(L(Z_B))$, i.e., $(p_B(x_k))_{k\in\mathbb{N}} \in L$ for every $x = (x_k)_{k\in\mathbb{N}} \in \mathfrak{B}$ and $\sup_{x\in\mathfrak{B}} \|(p_B(x_k))_{k\in\mathbb{N}}\| < +\infty$.*

We refer to [6] for df-spaces and properties of this class of locally convex spaces. Every DF-space of Grothendieck (in particular strong duals of Fréchet spaces) and LB-spaces belong to this class.

2.2 Lemma. *Let $(L, \|\,\|)$ satisfy (δ) and let Z be a df-space. Then for every $\mathfrak{B} \in \mathfrak{b}(L(Z))$, there is $B \in \mathfrak{b}(Z)$ such that $\mathfrak{B} \in \mathfrak{b}(L(Z_B))$*

The condition that $(L, \|\,\|)$ satisfies (δ) is needed in lemma 2.2. In fact if Z is any locally convex space and $(L, \|\,\|) = (c_o, \|\,\|_\infty)$ the fact that for every $\mathfrak{B} \in \mathfrak{b}(c_o(Z))$, there is $B \in \mathfrak{b}(Z)$ such that $\mathfrak{B} \in \mathfrak{b}(c_o(Z_B))$ implies that Z has the Mackey convergence condition (see e.g. [10]). There are locally convex spaces Z which do not satisfy the Mackey convergence condition.

We would like to recall -as it was done in [3]- that whenever $(L, \|\,\|)$ is a normal Banach sequence space satisfying property (ε), its dual space $(L', \|\,\|')$ coincides with the $\alpha-dual$ L^* and $(L'\|\,\|')$ has properties (β) and (δ).

2.3 Proposition. *Let* $(L, \|\,\|)$ *be a normal Banach sequence space which fulfils property* (ε) *and let* Z *be a locally convex space such that*
i) For every $\mathfrak{B} \in \mathfrak{b}(L(Z))$, *there is* $B \in \mathfrak{b}(Z)$ *with* $\mathfrak{B} \in \mathfrak{b}(L(Z_B))$.
ii) For every $u \in L'(Z'_b)$, *there is an absolutely convex equicontinuous set* $M \subset Z'$ *with* $u \in L'(Z'_M)$.
Then $L(Z)'_b$ *is canonically algebraically and topologically isomorphic to* $L'(Z'_b)$.

Remark. In particular i) and ii) are satisfied if either L satisfies (δ) and Z is a quasi-barrelled DF-space or L satisfies (ε) and Z is metrizable.

Proof. For every $k \in \mathbb{N}$, we denote $j_k : Z \to L(Z), x \to (\delta_{kj}x)_{j\in\mathbb{N}}$. Now we define $\varphi : L(Z)'_b \to L'(Z'_b), v \to (v \circ j_k)_{k\in\mathbb{N}}$

1. φ is well defined. (φ is clearly linear). Fix $v \in L(Z)'_b$. There must be $U \in \mathfrak{U}_o(Z)$ such that $|\langle v, x\rangle| \le \|(p_U(x_k))_{k\in\mathbb{N}}\|, \forall x \in L(Z)$. Take any $B \in \mathfrak{b}(Z)$. We denote $p_{B^o}(u) := \sup_{z\in B} |\langle u, z\rangle|, (u \in Z')$. We must show that $(p_{B^o}(v \circ j_k))_{k\in\mathbb{N}} \in L'$, that is, $\sum_{k\in\mathbb{N}} p_{B^o}(v \circ j_k)\alpha_k < +\infty$, for all $\alpha \in L$ with $\alpha_k > 0 (k \in \mathbb{N})$. Fix $n \in \mathbb{N}$. For every $z_1, \dots, z_n$ belonging to B we may write:

$$\sum_{k=1}^{n} |< v \circ j_k, z_k >|\alpha_k = \sum_{k=1}^{n} |< v \circ j_k, \alpha_k z_k >|$$

$$=(\text{for suitable } \beta_k \in \mathbb{K}, |\beta_k| = 1 (k \le n)) =$$

$$= \sum_{k=1}^{n} < v \circ j_k, \beta_k \alpha_k z_k >= v\left(\sum_{k=1}^{n} j_k(\beta_k \alpha_k z_k)\right) \le \|((p_U(\beta_k \alpha_k z_k))_{k\le n}, (0)_{k>n})\| \le \mu\|\alpha\|,$$

where $\mu > 0$ is such that $p_U(y) \le \mu$ for all $y \in B$. Consequently $\sum_{k=1}^{n} p_{B^o}(v \circ j_k)\alpha_k \le \|\alpha\|\mu$. Since n is arbitrary, the proof of **1.** is complete.

2. φ is continuous. Fix $C \in \mathfrak{b}(Z)$. We define the following bounded set in $L(Z)$: $\mathfrak{C} := \{\sum_{k=1}^{n} j_k(\beta_k \alpha_k z_k) : \beta_k \in \mathbb{K}, |\beta_k| = 1 (k \leq n), z_1, \ldots, z_n \in C, n \in \mathbb{N}$, and $\alpha = (\alpha_k)_{k \in \mathbb{N}} \in L$ with $||(\alpha_k)_{k \in \mathbb{N}}|| = 1\}$.

Take $v \in \mathfrak{C}^o$. Let us check that $||(p_{c^o}(v \circ j_k))_{k \in \mathbb{N}}||' \leq 1$ or equivalently that for every $n \in \mathbb{N}$, $||((p_{c^o}(v \circ j_k))_{k \leq n}, (0)_{k>n}||' \leq 1$. Fix $n \in \mathbb{N}$. On one hand

$$||((p_{c^o}(v \circ j_k))_{k \leq n}, (0)_{k>n})||' = \sup_{\substack{\alpha \in L \\ ||\alpha||=1}} \left| \sum_{k=1}^{n} p_{c^o}(v \circ j_k) \alpha_k \right|.$$

On the other hand for every $\alpha = (\alpha_k)_{k \in \mathbb{N}} \in L$ with $||\alpha|| = 1$ and every $z_1, \ldots, z_n \in C$, $\sum_{k=1}^{n} | < v \circ j_k, \alpha_k z_k > | =$ (for suitable $\beta_k \in \mathbb{K}, |\beta_k| = 1$ $(k \leq n)$) $=$

$$\sum_{k=1}^{n} < v \circ j_k, \beta_k \alpha_k z_k >=$$

$$v\left(\sum_{k=1}^{n} j_k(\beta_k \alpha_k z_k)\right).$$

Since $v \in \mathfrak{C}^o$, the definition of $\mathfrak{C}$ proves **2.**

3. The mapping

$$\psi : L'(Z'_b) \to L(Z)'_b$$
$$u = (u_k)_{k \in \mathbb{N}} \to \psi(u) : L(Z) \to \mathbb{K}$$
$$x = (x_k)_{k \in \mathbb{N}} \to \sum_{h \in \mathbb{N}} < u_k, x_k >$$

is well defined. Fix $x = (x_k)_{k \in \mathbb{N}} \in L(Z)$. By i), there must be $C \in \mathfrak{b}(Z)$ such that $z \in L(Z_c)$. Since $(p_{c^\circ}(u_k))_{k \in \mathbb{N}} \in L'$,

$$\sum_{k=1}^{\infty} | < u_k, x_k > | \leq \sum_{k=1}^{\infty} p_{c^o}(u_k) p_c(x_k) \leq ||(p_{c^o}(u_k))_{k \in \mathbb{N}}||' ||(p_c(x_k))_{k \in \mathbb{N}}||$$

that is, $\psi(u)$ is well defined ($\psi(u)$ is clearly linear). By ii), there is $U \in \mathfrak{U}_o(Z)$ such that $(p_{U^o}(u_k))_{k \in \mathbb{N}} \in L'$. Thus for every $x \in L(Z)$, we obtain

$$|\psi(u)(x)| = \sum_{k=1}^{\infty} | < u_k, x_k > | \leq \sum_{k=1}^{\infty} p_{U^o}(u_k) p_U(x_k) \leq ||(p_{U^o}(u_k))_{k \in \mathbb{N}}||' ||(p_U(x_k))_{k \in \mathbb{N}}||$$

that is, $\psi(u)$ is continuous and **3.** is established.

Clearly ψ is linear, $\Psi \circ \phi = 1_{L(X)'_b}$ and $\phi \circ \Psi = 1_{L'(X'_b)}$

4. ψ is continuous. Take $\mathfrak{B} \in \mathfrak{b}(L(Z))$. By i), there is $B \in \mathfrak{b}(Z)$ such that $\mathfrak{B} \in \mathfrak{b}(L(Z_B))$. In particular there is $\mu > 0$ with $\|(p_B(x_k))_{k\in\mathbb{N}}\| \leq \mu, (x = (x_k)_{k\in\mathbb{N}} \in \mathfrak{B})$. Thus $\psi(u) \in \mathfrak{B}^o$ for all $u \in L'(Z'_b)$ with $\|(p_{B^o}(u_k))_{k\in\mathbb{N}}\|' \leq \mu^{-1}$. Indeed for every $x = (x_k)_{k\in\mathbb{N}} \in \mathfrak{B}$, $| < \psi(u), x > | \leq \sum_{k=1}^{\infty} | < u_k, x_k > | \leq \sum_{k=1}^{\infty} p_{B^o}(u_k)p_B(x_k) \leq \mu\mu^{-1} = 1$. ∎

3. DUALITY

Let us first recall the definition of a general inductive limit of Moscatelli type.

Let $(L, \|\,\|)$ be a normal Banach sequence space, let Y and X be locally convex space, Y continuously included in X. For every $n \in \mathbb{N}$, the space $E_n := \prod_{k<n} X \times L((Y)_{k\geq n})$ has the obvious meaning and should be provided with the canonical product topology. Now we define the inductive limit of Moscatelli type w.r.t. $(L, \|\,\|), X, Y$ (and the continuous canonical inclusion $j : Y \to X$) as $E = \operatorname{ind}_{n\in\mathbb{N}} E_n$. (we refer to [8] for details).

Let $(L, \|\,\|)$ be a normal Banach sequence space, let Y and X be locally convex space, $f : Y \to X$ a continuous linear mapping and F the corresponding projective limit of Moscatelli type. For every sequence of subsets $(B_k)_{k\in\mathbb{N}}$ in $\mathfrak{b}(Y)$ and every subset $\mathfrak{B} \in \mathfrak{b}(X)$, which we shall always choose closed and absolutely convex, we define the space $F_{B(B_k)} := L(([B_k \cap f^{-1}(B)], p_{B_k \cap f^{-1}(B)})_{k\in\mathbb{N}})$ (compare with the definition in [3]). Here [A] means the linear span of A and p_A is the Minkowski functional of A.

The space $F_{B(B_k)}$ is continuously embedded in F $((B_k)_{k\in\mathbb{N}}$ in $\mathfrak{b}(Y), B \in \mathfrak{b}(X))$.

3.1 Proposition. *Let $(L, \|\,\|)$ be a normal Banach sequence space, let Y and X be locally convex spaces and $f : Y \to X$ a continuous linear mapping. Let F be the corresponding projective limit of Moscatelli type and let $F_{B(B_k)}$ be as above. If either X is a metrizable space or the space $(L, \|\,\|)$ satisfies (δ) and X is a df-space, then for every $\mathfrak{B} \in \mathfrak{b}(L(X))$, there are $B \in \mathfrak{b}(X)$ and $(B_k)_{k\in\mathbb{N}}$ in $\mathfrak{b}(Y)$ such that $\mathfrak{B} \in \mathfrak{b}(F_{B(B_k)})$. In particular, ind $(F_{B(B_k)} : B \in \mathfrak{b}(X), (B_k)_{k\in\mathbb{N}}$ in $\mathfrak{b}(Y))$ is the bornological space associated to F.*

Proof. Given $\mathfrak{B} \in \mathfrak{b}(F)$, we have $\tilde{f}(\mathfrak{B}) \in \mathfrak{b}(L(X))$ (cf.1.1). By lemma (2.1) or (2.2) we can find $\mathfrak{B} \in \mathfrak{b}(X)$ such that $\tilde{f}(\mathfrak{B}) \subset \{(x_k)_{k\in\mathbb{N}} \in L(X) : (p_B(x_k))_{k\in\mathbb{N}} \in L$ and $\|(p_B(x_k))_{k\in\mathbb{N}}\| \leq 1\}$. Define $B_k := \eta_k^{-1} pr_k(\mathfrak{B})(k \in \mathbb{N})$ where $(\eta_k)_{k\in\mathbb{N}} \in L$ with $\|(\eta_k)_{k\in\mathbb{N}}\| = 1$ and $\eta_k > 0, (k \in \mathbb{N})$. Thus $\mathfrak{B} \in \mathfrak{b}(L([B_k \cap f^{-1}(B)], p_{B_k \cap f^{-1}(B)})_{k\in\mathbb{N}})$ since for every $x = (x_k)_{k\in\mathbb{N}} \in \mathfrak{B}$, we may write:

$$p_{B_k \cap f^{-1}(B)}(x_k) = \max(p_{B_k}(x_k), p_{f^{-1}(B)}(x_k)) \leq \max(\eta_k, p_B(f(x_k)))(k \in \mathbb{N}). \blacksquare$$

3.2 Proposition. *Let $(L, \|\ \|)$ be a Normal Banach sequence space with property (ε), let Y and X be locally convex spaces and $f : Y \to X$ a continuous linear mapping with dense range. Let F be the corresponding projective limit of Moscatelli type. Let E be the inductive limit of Moscatelli type w.r.t. the duals $(L'\|\ \|'), X'_b, Y'_b$ and $f^t : X'_b \to Y'_b$ (that we shall always consider as an inclusion). If the following two conditions are satisfied:*
i) For every $\mathfrak{B} \in \mathfrak{b}(L(X))$ there is $B \in \mathfrak{b}(X)$ such that $\mathfrak{B} \in \mathfrak{b}(L(X_B))$.
ii) Foe every $u \in L'(X'_b)$ there is an absolutely convex X-equicontinuous set $M \subset X'$ such that $u \in L'(X'_M)$.

Then $F' = E$ algebraically and E is continuously embedded in F'_b.

Proof. By proposition (2.3), we have $F'_{n,b} = \prod_{k<n} Y'_b \times L'((X'_b)_{k\geq n}) = E_n (n \in \mathbb{N})$ algebraically and topologically. Besides, the continuous linear mapping:

$$g^t_n : F'_{n,b} = \prod_{k<n} Y'_b \times L'((X'_b)_{k\geq n}) \to F'_{n+1,b} = \prod_{k<n+1} Y'_b \times L'((X'_b)_{k\geq n+1})$$

$$v \to v \circ g_n$$

that is, $g^t_n(v) = v \circ g_n = ((v_k)_{k<n}, f^t(v_n), (v_k)_{k\geq n+1})(v \in F'_{n,b})$, coincides with the canonical inclusion $g^t_n : E_n \to E_{n+1} (n \in \mathbb{N})$.
But F is reduced (which is easily derived from the fact that f has dense range) and therefore by [7, 22.6(6)], we obtain $F' = \operatorname{ind}_{n\in\mathbb{N}} (F'_n, g^t_n) = E$ algebraically. Besides, the inclusion $E \subset F'_b$ is continuous because of the definition of the inductive limit topology.

The topological identity in the theorem above is rather delicate. We refer to [3] and [5] for the case of Banach spaces Y and X.

3.3 Proposition. *Let $(L, \|\ \|)$ be a normal Banach sequence space with property (ε), let Y and X be locally convex spaces, Y continuously included in X. Let E be the inductive limit of Moscatelli type w.r.t. $(L, \|\ \|), Y, X$ (and the continuous canonical inclusion $j : Y \to X$). Let F be the corresponding projective limit of Moscatelli type w.r.t. the duals $(L', \|\ \|'), X'_b, Y'_b$ and j^t. If the following two conditions are satisfied:*
i) For every $\mathfrak{B} \in \mathfrak{b}(L(Y))$ there is $B \in \mathfrak{b}(Y)$ such that $\mathfrak{B} \in \mathfrak{b}(L(Y_B))$.
ii) Foe every $u \in L'(Y'_b)$, there is an absolutely convex Y-equicontinuous set $M \subset Y'$ such that $u \in L'(Y'_M)$.
Then $E' = F$ albegraically, the inclusion $E'_b \subset F$ is continuous and $E'_b = F$ algebraically and topologically whenever E is regular.

Proof. By proposition (2.3), we have $E'_{n,b} = \prod_{k<n} X'_b \times L'((Y'_b)_{k\geq n}) = F_n (n \in \mathbb{N})$ algebraically and topologically. If we denote $j_n : E_n \to E_{n+1}, (x_k)_{k\in\mathbb{N}} \to ((x_k)_{k<n}, j(x_n),$

$(x_k)_{k>n})$ $(n \in \mathbb{N})$ we obtain that

$$j_n^t : E'_{n+1,b} = \prod_{k<n+1} X'_b \times L'((Y'_b)_{k\geq n+1}) \to E'_{n,b} = \prod_{k<n} X'_b \times L'((Y'_b)_{k\geq n})$$

$$v \to v \circ j_n$$

coincides with the canonical mapping from F_{n+1} to F_n $(n \in \mathbb{N})$.

Thus, by [7, 22.6.(4)], we have $E' = F$ (algebraically) and the inclusion $E'_b \subset F$ is continuous because of the definition of the projective limit topology. Besides, if E is regular, we have $E'_b = \operatorname{proj}_{n\in\mathbb{N}} (E'_{n,b}, f_n^t) = \operatorname{proj}_{n\in\mathbb{N}} F_n = F$; algebraically and topologically. ∎

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