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The Formation of Portfolio with Fuzzy Approach and Multi-objective Method

A Case Study on Stocks incorporated in LQ45

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Forming a portfolio in the investment process is a crucial component. It is because investors want maximum profit while expecting a minimum level of risk. The portfolio composition is inseparable from the weighting of each observed stock. In fact, mathematically, there are still problems when trying to fulfill the preferences that investors want. The research objective was the formation of a portfolio using a fuzzy approach and a multi-objective method. This model simultaneously maximized the return and risk of the prepared portfolio. The result was the formation of a portfolio with two categories, namely risk-seeking and risk-averse, equipped with a λ value of each method, the weight of each stock, the expected return, and risk. Parameter λ was the value obtained from selecting the risk level determined by the investor. Parameter λ was used to assess the level of risk and the expected return on the portfolio preparation. The last section compared the weights, expected return, and risk values of the two methods. As a result, investors in the risk seeker category have the potential to get higher expected returns when using the multi-objective method. In contrast, the fuzzy approach produces the possibility of a higher expected return for investors in the risk-averse category.

Keywords: Expected Return, Fuzzy Portfolio, Multi-objective Portfolio.

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1 Introduction

A portfolio is a collection of assets in the form of investments that individuals or Company institutions can own (Ta et al., 2020; Subekti et al., 2022; Pramono et al., 2022). The investment can be in the form of tangible assets and financial assets. Investments in tangible assets include gold, property, business capital, etc.(Jreisat, 2018; Sumer and Ozorhon, 2021). Meanwhile, investments in financial assets include stocks, deposits, bonds, mutual funds, etc (Maghyereh and Abdoh, 2020). Each share proportion in the portfolio is referred to as portfolio weight (Raihan and Saepudin, 2018). Forming a portfolio by minimizing risk and maximizing profit often encounters obstacles such as many shares being compared for purchase, investor preferences regarding investment objectives, and so on (Caviezel et al., 2012; Manik and Sukandar, 2021). In fact, one of the most popular topics in applied finance is portfolio selection, which is choosing the most suitable combination of securities to fulfill investors' goals (Huang, 2008).

On the other hand, investors' expectations regarding financial parameters based on portfolio decision-making are often vaguely stated. For example, an investor expects significant profits to exceed 30% of the total investment or controllable losses to be significantly less than 15% (Gupta et al., 2014). It indeed poses its challenges in terms of methodological in forming a portfolio that is under the preferences of investors.

In these conditions, the fuzzy set theory will be more helpful. It is more evident that modeling a portfolio selection will depend on the development results of fuzzy set theory. Furthermore, the advantages include theory fuzzy accommodating ambiguity and uncertainty and providing flexibility in the decision-making process by combining investor preferences and expert knowledge. Bellman and Zadeh (1970) define fuzzy scope decision-making by integrating fuzzy objectives and fuzzy constraints. Unlike classical set theory, there is no clear boundary between elements included or not included in the set in fuzzy set theory.

Another method of portfolio formation can be formed using multi-objective optimization. Multi-objective optimization can be viewed as a solution with considering various aspects. Duan (2007); Subekti and Kusumawati (2015); Septiano et al. (2019); Goli et al. (2019); García García et al. (2020); Yu et al. (2021) state that the preparation of a portfolio with multi-objective optimization is based on maximizing the expected return and minimizing the risk obtained. Maximizing expected returns and minimizing risk are done simultaneously. This method gives investors a choice in allocating investment funds based on their respective characteristics. The characteristics can be included in the risk-averse or risk-seeking category.

This paper examined the formation of a portfolio with a fuzzy approach and a multi-objective method by considering the bi-objective portfolio optimization model based on the mean-variance framework proposed by Markowitz (1952). The model simultaneously maximized profits and minimized losses on the observed stock. Some relevant references for the fuzzy framework of portfolio selection using the mean-variance model were Ramaswamy (1998); Parra et al. (2001); Zhang and Nie (2005); Bilbao-Terol et al. (2006). Thus, the formulation of a bi-objective fuzzy portfolio selection problem was based on the level of vague investor aspirations regarding portfolio returns and risks to determine a satisfying portfolio selection strategy (Subekti and Kusumawati, 2015). It took into account that an investor determined preferences based on their experience and knowledge so far. Furthermore, the final part compared the performance of the fuzzy portfolio according to the investors' characteristics. The grouping of investor categories is done so that investors can directly use the results obtained according to the type of investment style.

The main contributions in this research consisted of 1) simulating real-time data in portfolio arrangement with a fuzzy and multi-objective approach with several investor-type criteria, and 2) comparing the performance results of the fuzzy and approach methods, multi-objective optimization was seen from the weight, expected return, and risk obtained.

2 Method

2.1 Fuzzy Portfolio Selection Model

This model simultaneously maximized portfolio returns ($f_1(x)$) while minimizing portfolio risk ($f_2(x)$) which was formulated with (Almahdi and Yang, 2017; Kalayci et al., 2019):

$$\max(f_1(x)) = \sum_{i=1}^{n} r_i x_i \tag{1}$$

$$\min(f_2(x)) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j \tag{2}$$

With constraint function

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

and

$$x_i \geq 0,\ \ i = 1, 2, 3, ..., n \tag{4}$$

with $r_i = E[R_i]$, $\sigma_{ij} = E[(R_i - r_i)(R_j - r_j)]$ The linear membership function of the expected return of the portfolio is defined as follows:

$$\mu_{f_1}(x) = \begin{cases} 1 & ,\text{if} \quad f_1(x) \geq f_1^R \\ \frac{f_1(x) - f_1^L}{f_1^R - f_1^L} & ,\text{if} \quad f_1^L < f_1(x) < f_1^R \\ 0 & ,\text{if} \quad f_1(x) \leq f_1^L \end{cases} \tag{5}$$

with f_1^L was the lowest lower limit, and f_1^R was the highest upper bound of a portfolio return that investors wanted (see Figure 1).

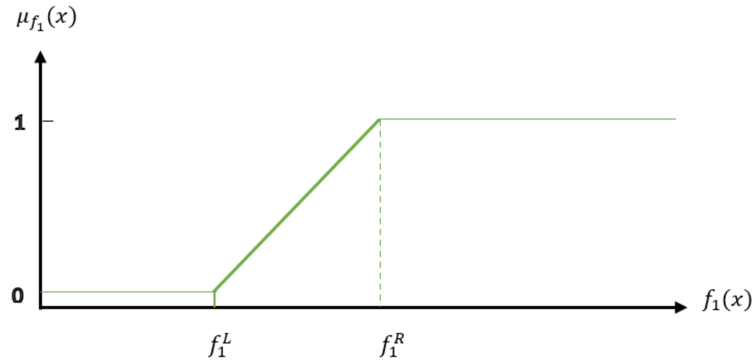

Figure 1: The purpose of the fuzzy portfolio return

Meanwhile, the linear membership function of portfolio risk is defined as follows (Gupta et al., 2014; Ciavolino and Calcagnì, 2016):

$$\mu_{f_2}(x) = \begin{cases} 1 & \text{, if} \quad f_2(x) \geq f_2^L \\ \frac{f_2^R - f_2(x)}{f_2^R - f_2^L} & \text{, if} \quad f_2^L < f_2(x) < f_2^R \\ 0 & \text{, if} \quad f_2(x) \leq f_2^R \end{cases} \tag{6}$$

with f_2^R was the lowest lower limit, and f_2^L was the highest upper bound of a portfolio risk that investors wanted (see Figure 2).

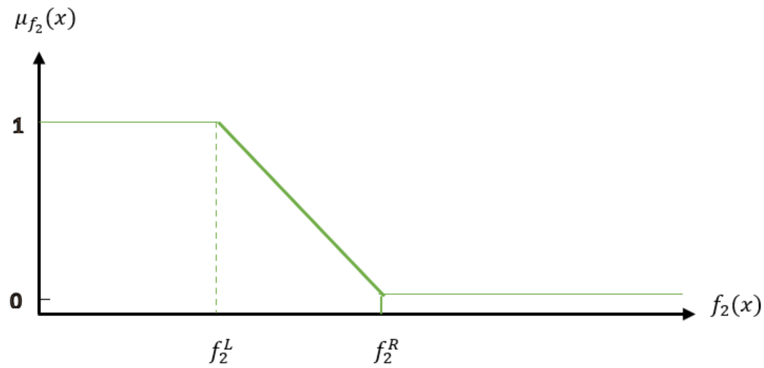

Figure 2: The purpose of fuzzy portfolio risk

It was based on the Bellman and Zadeh (1970) maximization approach and fuzzy membership functions. The fuzzy bi-objective optimization model for the portfolio selection problem is formulated as follows:

$$\max \lambda$$

Constraint function

$$\lambda \leq \mu_{f_1}(x),$$

$$\lambda \leq \mu_{f_2}(x),$$
$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0,\ i = 1, 2, 3, ..., n$$
$$0 \leq \lambda \leq 1$$

2.2 Fuzzy Interactive Approach Stages

The stages of solving the fuzzy interactive approach to the problems above are as follows (Gupta et al., 2014):

1. We are developing a mathematical model that is return and covariance variance matrix.
2. Solving the max $(f_1(x))$ and $\min(f_2(x))$ problems as a single-purpose problem in terms of the return and risk objective functions, taking into account the constraint functions if the solution was under the stage, so it should be stopped, if not, continued to step 3.
3. Evaluating the objective function of the solution obtained and determining the worst lower limit (f_L^1) and the best upper limit (f_R^1) for return purposes; as well as the best lower limit (f_L^2) and the worst upper bound (f_R^2) for risk purposes.
4. We are determining the linear membership function for return and risk.
5. Maximizing λ with constraint function:

$$\lambda < \mu_{f_1}(x),$$
$$\lambda < \mu_{f_2}(x),$$
$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0, i = 1, 2, 3, ..., n,$$
$$0 < \lambda < 1$$

Then, the processes were completed. The process was stopped if the settlement results were in line with investors' expectations. Otherwise, both objective functions were re-evaluated. The role of λ in this method is as the objective function to be maximized. In the return objective function, the current worst lower bound with a new objective value was compared. If the new value was higher than the worst lower limit, it was set as the new limit bottom; otherwise, it used the old value. On the other hand, the current worst upper bound is compared to the new goal value for risk purposes. If the new value was lower than the worst upper limit, it was set as the new upper limit. If not, an old value was used. If there were no change in the boundary of the two objective functions, then it would be stopped. If not, it should be continued to stage 4 and repeat the solution process.

2.3 Multi-objective Portfolio Selection Model

The method employed a multi-objective portfolio and principally aimed to maximize return and minimize risk. Maximizing $R_p = r^T w$ and minimizing $\sigma_p = w^T \sum w$ with constraint function $\mathbf{1}^T\mathbf{w} = 1$ (Ruiz-Torrubiano and Suárez, 2015; Hoyyi and Ispriyanti, 2015; Seyedhosseini et al., 2016). It was equivalent to minimizing $(-r^T w, w^T \sum w)$ with $\mathbf{1}^T\mathbf{w} = 1$. Then, adding the two weighting coefficients $a_1 > 0$ and $a_2 > 0$ to get the minimum equation of $-r^T w + w^T \sum w$.

Furthermore, a multi-objective optimization was solved using the Lagrange function as follows:

$$L = -r + 2kw^T \sum w + \lambda(\mathbf{1}^T w - 1) \tag{7}$$

In finding the optimal solution of w, equation 7 was derived concerning w and then equated to zero.

$$\frac{dL}{dw} = -\mathbf{r} + 2k \sum \mathbf{w} + \lambda \mathbf{1}_p = 0 \tag{8}$$

By transposing the results of equation 8, it is obtained:

$$w = \frac{1}{2k}(\sum)^{-1}(\mathbf{r} - \lambda \mathbf{1}_p) \tag{9}$$

Substituting equation 9 into equation $1_p^T w = 1$ which obtained:

$$\lambda = \frac{\mathbf{1}_p^T(\sum)^{-1} r - 1}{\mathbf{1}_p^T(\sum)^{-1}\mathbf{1}_p} \tag{10}$$

The steps of the multi-objective portfolio method included (Di Asih and Purbowati, 2009; Pradana et al., 2015):

1. Determining the mean and variance of observed stock returns. The mean and variance of returns are obtained from daily stock data observed over a certain period.

2. Determining the variance and covariance matrix. The variance-covariance matrix is prepared based on the variance value and covariance of return on assets.

3. Generating various k-values. The value of k is determined by gradation to get a simulation of the investor category.

4. Specifying the λ parameter. The lambda value is obtained using Equation 10.

5. Determining the weight of each stock that depended on the k and λ values. Interpretation of simulation results.

6. Determining the expected return and risk. This is obtained using the $R_p = r^T w$ and $\sigma_p = w^T$ equations after the weight of each portfolio is obtained.

The role of λ is as a Lagrange multiplier, which is then used to determine the weighting of the formed portfolio. In the end, get the return and risk of the existing portfolio.

3 Results and Discussion

3.1 Data

The data used were stock data from PTBA.JK, MNCN.JK, EXCL.JK, BMRI.JK, and ADHI.JK in LQ-45. The data period was daily stock data for the entire eight months. The selection of stock members of the LQ-45 is due to the most active and liquid stocks on the daily transfer market. The following in table 1 was the mean return data and mean-variance of the data used:

Table 1: Mean Return and Variance of Stock Data Owned

No.	Stocks	Mean Return	Variance
1	PTBA.JK	0.000392381	0.000511556
2	MNCN.JK	0.000288736	0.000732135
3	EXCL.JK	0.000722567	0.000706930
4	BMRI.JK	0.000327829	0.000450177
5	ADHI.JK	0.001529785	0.001341421

Meanwhile, the variance and covariance matrices are as follows:

Table 2: Matrix of variance and covariance of Stock Data owned

Stocks	PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK
PTBA.JK	0.000514	0.000220	0.000225	0.000164	0.000284
MNCN.JK	0.000220	0.000735	0.000272	0.000211	0.000466
EXCL.JK	0.000225	0.000272	0.000710	0.000184	0.000270
BMRI.JK	0.000164	0.000211	0.000184	0.000452	0.000260
ADHI.JK	0.000284	0.000466	0.000270	0.000260	0.001347

The value of the covariance variance matrix in Table 2 showed the relationship between stocks' positive values. It meant that if one stock went up, other stocks also tended to increase and vice versa.

3.2 Portfolio Development with Fuzzy

Step 1

We formulate the model according to equations 1 and 2, constraint functions 3 and 4.

$$\begin{aligned}\max\ f_1(x) &= 0.000392381x_1 + 0.000288736x_2 + 0.000722567x_3 \\ &\quad +0.000327829x_4 + 0.001529785x_5\end{aligned}$$

and

$$\begin{aligned}\min\ f_2(x) &= 0.000514x_1x_1 + 0.000735x_2x_2 + 0.000710x_3x_3\\ &+0.000452x_4x_4 + 0.001347x_5x_5 + 0.000220x_1x_2\\ &+0.000223x_1x_3 + 0.000164x_1x_4 + 0.000284x_1x_5\\ &+0.000272x_2x_3 + 0.000211x_2x_4 + 0.000466x_2x_5\\ &+0.000184x_3x_4 + 0.000270x_3x_5 + 0.000260x_4x_5\end{aligned}$$

Constraint function:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$
$$x_i \geq 0,\ i = 1, 2, ..., 5.$$

Step 2

Determining the worst lower (biggest) limit and the best upper (smallest) limit for the return and risk functions, respectively, the problem solved was as a single objective as follows:

Return Function

$$\begin{aligned}\max\ f_1(x) &= 0.000392381x_1 + 0.000288736x_2 + 0.000722567x_3\\ &+0.000327829x_4 + 0.001529785x_5\end{aligned}$$

Constraint function:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$
$$x_i \geq 0,\ i = 1, 2, ..., 5.$$

Obtained solution (x^1) is presented in table 3:

Risk Function

$$\begin{aligned}\min\ f_2(x) &= 0.000514x_1x_1 + 0.000735x_2x_2 + 0.000710x_3x_3\\ &+0.000452x_4x_4 + 0.001347x_5x_5 + 0.000220x_1x_2\\ &+0.000223x_1x_3 + 0.000164x_1x_4 + 0.000284x_1x_5\\ &+0.000272x_2x_3 + 0.000211x_2x_4 + 0.000466x_2x_5\\ &+0.000184x_3x_4 + 0.000270x_3x_5 + 0.000260x_4x_5\end{aligned}$$

Constraint function:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$
$$x_i \geq 0,\ i = 1, 2, ..., 5.$$

Obtained solution (x^2) is presented in table 3:

Table 3: Proportion of assets in the portfolio acquired for a single purpose

	Allocation				
	PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK
x^1	0.0	0.0	0.0	0.0	1.0
x^2	0.2790079	0.1528888	0.1759434	0.3385981	0.05356192

Step 3
The evaluation of the two objective functions on the solution obtained, namely x^1 and x^2. The function value of objectives is presented in table 4. Thus, the worst lower (biggest) limit and the upper limit (smallest), the best, of the two objective functions are obtained as follows:

$$0.000289 \leq f_1(x) \leq 0.00153$$
$$0.00045 \leq f_2(x) \leq 0.001341$$

Table 4: Return and risk objective function values in the two solutions obtained

	x^1	x^2
Return $(f_1(x))$	0.001530	0.000289
Risk $(f_1(x))$	0.001341	0.000450

Step 4
The formation of membership functions for return and risk is as follows:
The linear membership function of the portfolio's expected return is:

$$\mu f_1(x) = \begin{cases} 1 & \text{if,} \quad f_1(x) \geq 0.001530 \\ \frac{f_1(x)-0.000289}{0.001530-0.000289} & \text{if,} \quad 0.000289 < f_1(x) < 0.001530 \\ 0 & \text{if,} \quad f_1(x) \leq 0.000289 \end{cases}$$

The linear membership function of portfolio risk is:

$$\mu f_2(x) = \begin{cases} 1 & \text{if,} \quad f_2(x) \leq 0.000450 \\ \frac{0.001341-f_2(x)}{0.001341-0.000450} & \text{if,} \quad 0.000450 < f_2(x) < 0.001341 \\ 0 & \text{if,} \quad f_2(x) \geq 0.001341 \end{cases}$$

Step 5
Formulating the model as in step 5 is presented as follows:

$$\max \lambda$$

Constraint function

$$\begin{aligned}&0.000392381x_1 + 0.000288736x_2 + 0.000722567x_3\\&+0.000327829x_4 + 0.001529785x_5 - 0.001530\lambda \geq 0.000289\end{aligned}$$

$$\begin{aligned}&0.000514x_1x_1 + 0.000735x_2x_2 + 0.000710x_3x_3\\&+0.000452x_4x_4 + 0.001347x_5x_5 + 0.000220x_1x_2\\&+0.000223x_1x_3 + 0.000164x_1x_4 + 0.000284x_1x_5\\&+0.000272x_2x_3 + 0.000211x_2x_4 + 0.000466x_2x_5\\&+0.000184x_3x_4 + 0.000270x_3x_5 + 0.000260x_4x_5\\&+0.5335\lambda \leq 0.001341\end{aligned}$$

$$\begin{gathered}x_1 + x_2 + x_3 + x_4 + x_5 = 1\\x_i \geq 0,\ i = 1, 2, ..., 5.\\0 \leq \lambda \leq 1\end{gathered}$$

The computational results presented in tables 5 and 6 presented the proportion of assets in the portfolio for each share obtained.

Table 5: Summary of portfolio selection results

λ	Return $(f_1(x))$	Risk $(f_1(x))$
0.6351346	0.001260756	0.0007974948

Table 6: Proportion of assets in the portfolio with Fuzzy

Allocation				
PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK
0.0000001	0.00000002	0.3332791	0.00000007	0.6667207

Table 5 shows the expected return and risk values from the iteration results with the fuzzy stage first. Table 6 is the weighting for each share used in the portfolio preparation. The value obtained stopped if it matched the investors' preferences and goals. Suppose the investor was not satisfied with the solution obtained and would like to improve it further. As desired by investors, individual goals, namely return, can be increased; however, due to the nature of the multi-objective problem, improvement in one goal could produce effects detrimental to other purposes. Therefore, the researchers could modify the obtained solution depending on the investor's preference for both objectives. In this process, the lower and upper limits and the aspiration level of the selected objective function were modified.

3.2.1 Investor Risk Seeking Category with Fuzzy Portfolio

The type of investor in the risk-seeking category occurred when the solutions offered in table 6 did not match investor preferences. Thus, further iterations were needed regarding the expected return selection simulation under the wishes and goals of an investor. In this case, the expected return value was taken close to the maximum from the return data owned. The selection of the lower limit of the expected return was as much as ten parts. Next, a simulation was given, and the λ, value was calculated return, risk, and weight of each stock observed for each portfolio.

The following is a simulation result of the proportion of assets in the portfolio obtained by varying the lower limit of the expected return (see Table 7 and 8).

Table 7: λ value, Return, and Risk of Each Portfolio

	Boundary	λ	Return	Risk
			$f_1(x)$	$f_2(x)$
Portfolio 1	$0.001116102 \leq f_1(x) \leq 0.001529$	0.2158558	0.001446361	0.001140539
Portfolio 2	$0.001157470 \leq f_1(x) \leq 0.001529$	0.1933457	0.001453289	0.001156449
Portfolio 3	$0.001198839 \leq f_1(x) \leq 0.001529$	0.1710450	0.001460538	0.001173336
Portfolio 4	$0.001240207 \leq f_1(x) \leq 0.001529$	0.1489439	0.001468091	0.001191192
Portfolio 5	$0.001281575 \leq f_1(x) \leq 0.001529$	0.1270314	0.001475933	0.001210012
Portfolio 6	$0.001322943 \leq f_1(x) \leq 0.001529$	0.1052974	0.001484048	0.001229788
Portfolio 7	$0.001364312 \leq f_1(x) \leq 0.001529$	0.8373191e-01	0.001492422	0.001250517
Portfolio 8	$0.001405680 \leq f_1(x) \leq 0.001529$	0.6232695e-01	0.00150104	0.001272193
Portfolio 9	$0.001447048 \leq f_1(x) \leq 0.001529$	0.4107344e-01	0.00150989	0.001294811
Portfolio 10	$0.001488417 \leq f_1(x) \leq 0.001529$	0.4086077e-01	0.001509565	0.001293973

Table 8: Proportion of Assets in Portfolio with Fuzzy Risk Seeking Category

	Allocation				
	PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK
Portfolio 1	0.3322335e-08	0.2077097e-08	0.1033470	0.1371295e-08	0.8966530
Portfolio 2	0.2075730e-08	0.1426042e-08	0.9476508e-01	0.000000	0.9052349
Portfolio 3	0.000000	0.000000	0.8578490e-01	0.000000	0.9142151
Portfolio 4	0.6797864e-08	0.4297187e-08	0.7642770e-01	0.2809224e-08	0.9235723
Portfolio 5	0.5829635e-08	0.4165660e-08	0.6671311e-01	0.2247406e-08	0.9332869
Portfolio 6	0.1739046e-08	0.1296429e-08	0.5666008e-01	0.000000	0.9433399
Portfolio 7	0.3487255e-08	0.2618113e-08	0.4628635e-01	0.2352754e-08	0.9537136
Portfolio 8	0.000000	0.000000	0.3560967e-01	0.000000	0.9643903
Portfolio 9	0.6934179e-08	0.1044837e-07	0.2464595e-01	0.3828818e-08	0.9753540
Portfolio 10	0.1142187e-08	0.2254951e-08	0.2504903e-01	0.000000	0.9749510

The portfolios that have been compiled were choices to adjust investors' desire to make a profit. The expected profit was represented by $f_1(x)$ and the risks that might occur were represented by $f_2(x)$. For each portfolio was equipped with a weight and λ value for each stock analyzed. Indeed, more portfolios on offer would allow flexibility for investors to diversify investment assets according to their respective objectives.

3.2.2 Risk-Averse Investor Category with Fuzzy Portfolio

The case of investors with the risk-averse type was the medium criteria risk-taker. The simulation was presented with selecting risks taken from the observed stocks with the middle category. The simulation used ten data intervals to see the movement of the weights value of λ, expected returns, and risks from the compiled portfolio.

The following is the simulation result of the proportion of investment assets in the built portfolio with risk-averse criteria (see Table 9 and 10). The simulation results for

Table 9: λ Value, Return, and Risk of Each Portfolio

	Boundary	λ	Return	Risk
			$f_1(x)$	$f_2(x)$
Portfolio 1	$0.0007472583 \leq f_2(x) \leq 0.0010443397$	0.4551905	0.001446361	0.001140539
Portfolio 2	$0.0007769664 \leq f_2(x) \leq 0.0010443397$	0.4298995	0.001453289	0.001156449
Portfolio 3	$0.0008066746 \leq f_2(x) \leq 0.0010443397$	0.4049451	0.001460538	0.001173336
Portfolio 4	$0.0008363827 \leq f_2(x) \leq 0.0010443397$	0.3803143	0.001468091	0.001191192
Portfolio 5	$0.0008660909 \leq f_2(x) \leq 0.0010443397$	0.3559938	0.001475933	0.001210012
Portfolio 6	$0.0008957990 \leq f_2(x) \leq 0.0010443397$	0.3319705	0.001484048	0.001229788
Portfolio 7	$0.0009255071 \leq f_2(x) \leq 0.0010443397$	0.3082313	0.001492422	0.001250517
Portfolio 8	$0.0009552153 \leq f_2(x) \leq 0.0010443397$	0.2847629	0.00150104	0.001272193
Portfolio 9	$0.0009849234 \leq f_2(x) \leq 0.0010443397$	0.2615525	0.00150989	0.001294811
Portfolio 10	$0.0010146316 \leq f_2(x) \leq 0.0010443397$	0.2385876	0.001509565	0.001293973

the risk-averse investor category showed that the λ value decreases with an increased risk that might be borne. In this case, ADHI.JK stock weight tended to dominate while the other stocks fluctuated.

3.3 Multi-objective Portfolio Preparation

The preparation of a portfolio with a multi-objective approach was also divided into two categories of investor characteristics. The first characteristic was risk-seeking, marked by the relatively small k-value because they tended to dare to take risks in investing k-value for the risk-seeking category. It ranged from $0.001 \leq k \leq 1$, which was divided into ten parts to be used in the preparation of the portfolio. The second characteristic was risk-averse, which was moderate in risk-taking in investment. k-values in this category were in the interval $2 \leq k \leq 10$, divided into ten parts.

Table 10: Proportion of Assets in Portfolio with Fuzzy Risk-Averse Category

	Allocation				
	PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK
Portfolio 1	0.000000	0.000000	0.1621924	0.000000	0.8378076
Portfolio 2	0.000000	0.000000	0.1588807	0.000000	0.8411193
Portfolio 3	0.000000	0.000000	0.1549313	0.000000	0.8450687
Portfolio 4	0.000000	0.000000	0.1503687	0.000000	0.8496313
Portfolio 5	0.1060086e-07	0.5286434e-08	0.1452176	0.1356185e-08	0.8547824
Portfolio 6	0.7888612e-08	0.3849293e-08	0.1395033	0.000000	0.8604967
Portfolio 7	0.4257331e-08	0.1985612e-08	0.1332507	0.000000	0.8667493
Portfolio 8	0.1629161e-07	0.7621452e-08	0.1264847	0.1492405e-08	0.8735153
Portfolio 9	0.1764430e-08	0.000000	0.1192296	0.000000	0.8807704
Portfolio 10	0.5364400e-08	0.3351487e-08	0.1115093	0.2647106e-08	0.8884907

3.3.1 Categories of Investors with Multi-objective Risk Seeking

Formation of a risk-seeking portfolio began by dividing the value of $0.001 \leq k \leq 1$ into ten parts, then specified the parameter values of λ, weight, expected return, and risk for each portfolio. The following table 11 is the k-value used and λ value obtained:

Table 11: k and λ values in Risk Seeking Category

No	k Selected	Value λ
1	0.0010000	0.0003973873
2	0.1108889	0.0003337174
3	0.2207778	0.0002700474
4	0.3306667	0.0002063775
5	0.4405556	0.0001427076
6	0.5504444	7.903767e-05
7	0.6603333	1.536775e-05
8	0.7702222	-4.830218e-05
9	0.8801111	-0.0001119721
10	0.9900000	-0.000175642

Next, the k and λ values calculated each developed portfolio's weight, expected return, and risk.

The simulation results used a multi-objective portfolio approach with the risk-seeking category. The greater the k-value, the value of expected return and risk of each portfolio decayed slowly (see Table 12). Indeed, this condition made it easier for investors to choose the portfolio obtained in determining the allocation of investment according to preference.

Table 12: Proportion of Assets with a Multi-objective Approach to the Risk-Seeking Portfolio Category

	Allocation						
	PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK	Expected Return	Risk
Portfolio 1	-197.4461	-428.1188	296.9017	-273.6603	603.3235	0.8466854	423.144
Portfolio 2	-1.470019	-3.732272	2.832224	-2.065231	5.435298	0.008029818	0.03470185
Portfolio 3	-0.5823572	-1.8100381	1.5002534	-0.8350567	2.7271986	0.004231176	0.00897085
Portfolio 4	-0.2846802	-1.1654179	1.0535778	-0.4225185	1.8190388	0.002957304	0.004159667
Portfolio 5	-0.1355039	-0.8423763	0.8297331	-0.2157812	1.3639283	0.002318922	0.002469852
Portfolio 6	-0.04588967	-0.64831653	0.69526361	-0.09158859	1.09053118	0.001935429	0.001686266
Portfolio 7	0.013898368	-0.518845404	0.605549397	-0.008730812	0.908128452	0.001679573	0.001260125
Portfolio 8	0.05662629	-0.42631800	0.54143454	0.05048405	0.77777313	0.001496724	0.001002974
Portfolio 9	0.08868436	-0.35689617	0.49333018	0.09491201	0.67996961	0.001359536	0.0008359784
Portfolio 10	0.1136256	-0.3028858	0.4559049	0.1294771	0.6038782	0.001252802	0.0007214366

3.3.2 Risk-Averse Investor Category with Multi-objective

The formation of a risk-averse portfolio began by dividing the value of $2 \leq k \leq 10$ into ten parts, then specified the parameter values of λ, weight, expected return, and risk for each portfolio. The following table 13 shows the k and λ values which are obtained:

Table 13: k and λ values of the Risk-Averse Category

No	k Selected	Value λ
1	2.000000	-0.0007608388
2	2.888889	-0.001275863
3	3.777778	-0.001790888
4	4.666667	-0.002305913
5	5.555556	-0.002820937
6	6.444444	-0.003335962
7	7.333333	-0.003850987
8	8.222222	-0.004366011
9	9.111111	-0.004881036
10	10.000000	-0.005396061

Next, with the k and λ values, each developed portfolio calculated the weight, expected return, and risk.

In the case of the risk-averse investor category, the weight of each stock tended to be higher equally. However, it still provided an opportunity for a short sale in some of the portfolios presented. It can be seen in the risk-seeking case that MNCN.JK shares weighted lower compared to other observed stocks. Meanwhile, the shares of BMRI.JK had the highest weight. In this type of stock, there was no joint short sale with EXCL.JK, PTBA.JK and ADHI.JK shares, which were marked with a positive weight value. In the case of MNCN.JK shares, there was a short sale in portfolios 1 and 2 (see Table 14).

3.4 Discussion and Conclusion

In this section, the researchers compared portfolio selection with fuzzy and multi-objective approaches for two types of investors, i.e., risk-seeking and risk-averse. Some of the aspects presented in the comparison were the respective weights of stocks, expected returns, and risks in each portfolio.

3.4.1 Weighting in the Risk Seeking Category

This section would compare the weights and properties of each observed stock. Furthermore, it would be analyzed regarding the characteristics of the two methods offered. Figures 3 and 4 compare the resulting weights of portfolio formation with a fuzzy and multi-objective approach.

Table 14: Proportion of Assets with a Multi-objective Approach to the Risk-Averse Portfolio Category

	Allocation						
	PTBA.JK	MNCN.JK	EXCL.JK	BMRI.JK	ADHI.JK	Expected Return	Risk
Portfolio 1	0.21450291	-0.08443583	0.30453469	0.26927874	0.29611948	0.0008211104	0.0003954873
Portfolio 2	0.24492744	-0.01855144	0.25888154	0.31144284	0.20329962	0.0006909124	0.0003404035
Portfolio 3	0.26103455	0.01632853	0.23471222	0.33376501	0.15415970	0.000621984	0.0003193507
Portfolio 4	0.27100561	0.03792089	0.21975026	0.34758350	0.12373974	0.000579314	0.0003091314
Portfolio 5	0.27778594	0.05260369	0.20957613	0.35698007	0.10305417	0.0005502984	0.0003034112
Portfolio 6	0.28269583	0.06323607	0.20220865	0.36378448	0.08807497	0.0005292872	0.00029989
Portfolio 7	0.28641544	0.07129090	0.19662723	0.36893934	0.07672709	0.0005133695	0.0002975698
Portfolio 8	0.28933081	0.07760415	0.19225260	0.37297963	0.06783280	0.0005008935	0.0002959604
Portfolio 9	0.29167733	0.08268554	0.18873156	0.37623158	0.06067399	0.0004908519	0.0002947987
Portfolio 10	0.29360669	0.08686358	0.18583649	0.37890540	0.05478785	0.0004825955	0.0002939328

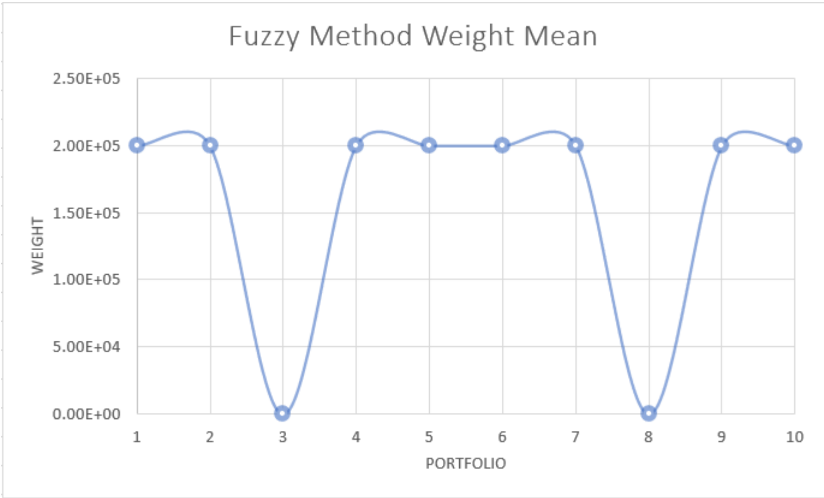

Figure 3: Portfolio Weight with Fuzzy Method

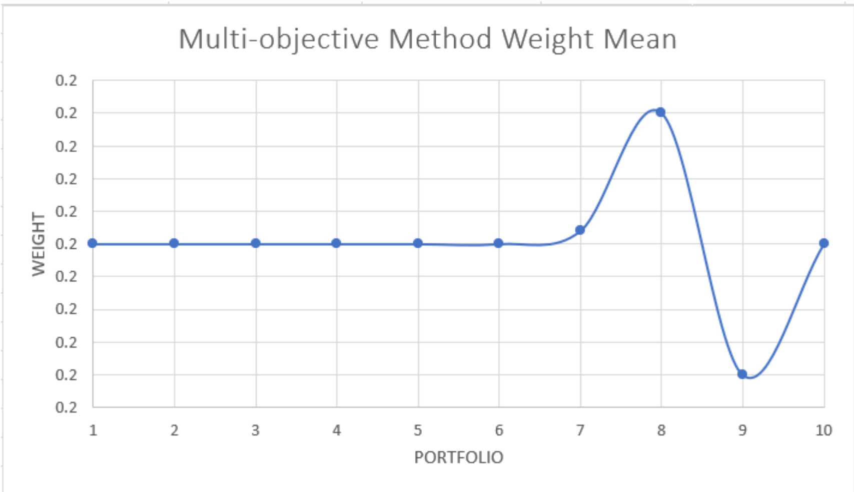

Figure 4: Portfolio Weight with Multi-objective Method

The average weight used in the fuzzy method descends on forming a portfolio third and fourth. Meanwhile, the other portfolios were almost consistent with relatively close numbers. The multi-objective method was only on the eighth and ninth portfolios, which experienced a difference in the mean distribution with other portfolios.

3.4.2 Expected Return and Risk in the Risk Seeking Category

Expected return and risk were fundamental pillars for investors. Therefore, the expected return and risk values of each method needed to be seen and scrutinized. Figures 5 and

6 compare the expected return and risk values from the fuzzy approach and the multi-objective method.

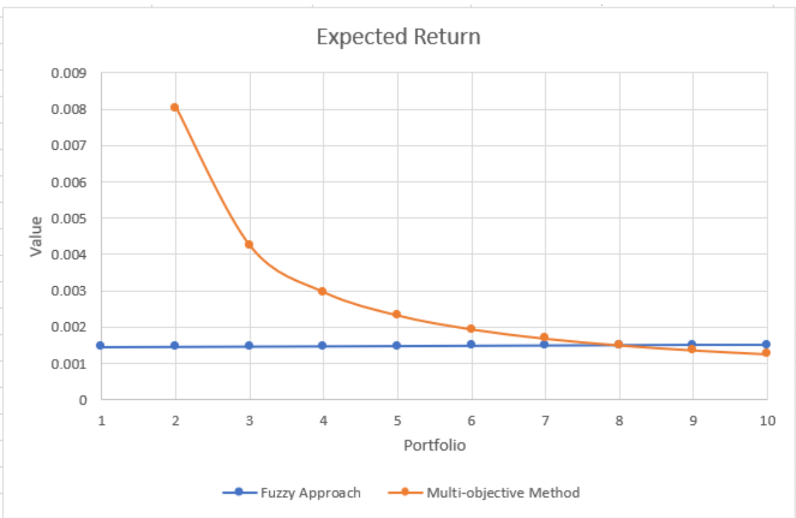

Figure 5: Expected Return with Fuzzy Approach and Multi-objective Method

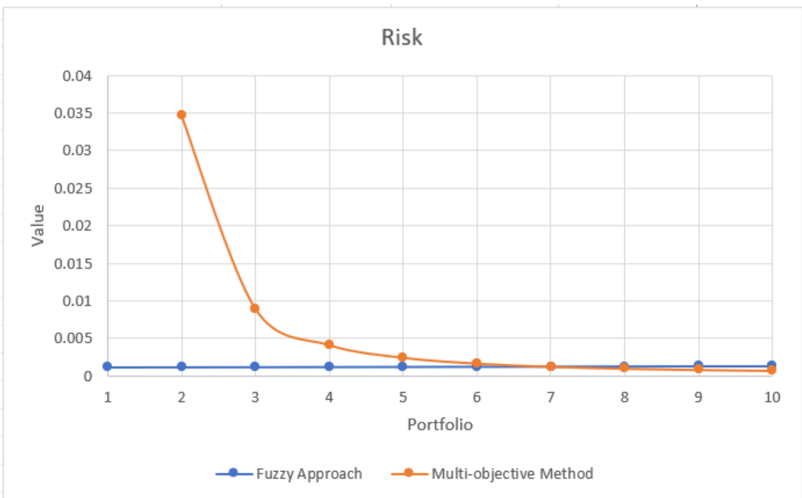

Figure 6: Risk with Fuzzy Approach and Multi-objective Method

Both methods had the same expected return intersection in the eighth portfolio. It could be seen from the intersection of the two lines formed. It showed the formation of the eighth portfolio with profit expectations of the same magnitude as both methods. The multi-objective method tended to have a higher expected return and had a declining pattern. Meanwhile, the expected return value tended to be stable with the fuzzy approach and increased slowly.

The risk in the risk-seeking category for the two methods intersected in portfolio 7.

It meant that in the 7th portfolio, both the fuzzy approach and the multi-objective method had the same level of risk. Meanwhile, in the 8th to 10th portfolio, the risk of the multi-objective method was under the portfolio formed by using a fuzzy approach.

3.4.3 Weighting in the Risk Averse Category

The subsequent discussion was the comparison of the average weight of the shares resulting from the formation portfolio with two methods in the risk-averse category. Figures 7 and 8 are visualizations of the comparison of the average weights of each observed stock.

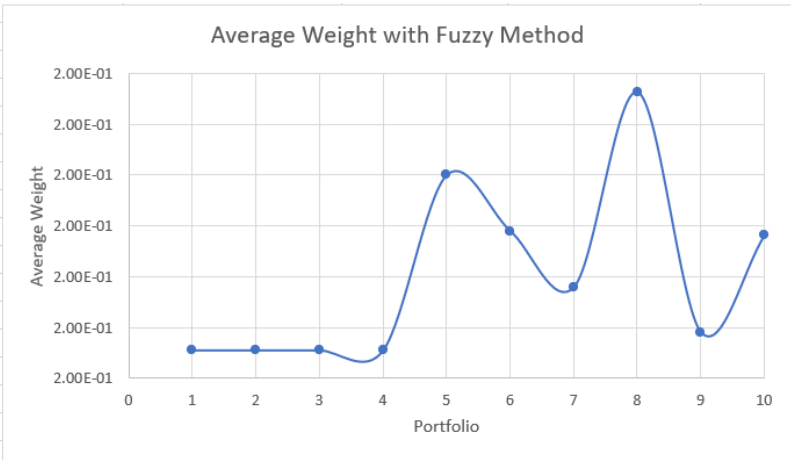

Figure 7: Portfolio Weight with Fuzzy Method

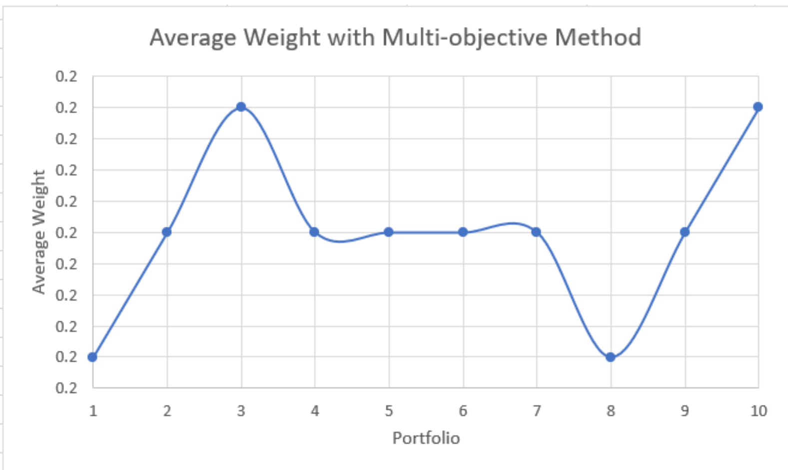

Figure 8: Portfolio Weights with Multi-objective Method

Through the Fuzzy method, the formation of the average weight on the observed stocks tended to be stable in the 1st- 4th portfolios. However, it was more dynamic and fluctuated on the 5th – 10th portfolio. The highest average portfolio weight occurred in the eighth portfolio. It was slightly different from using the multi-objective method. This method tended to be stable in the 4th- 7th portfolio. Meanwhile, the 1st- 3rd and 8th- 10th portfolios tended to be stable up and down. The highest average weight occurred in the 5th and 10th portfolios; meanwhile, the lowest was in the 1st and 8th portfolios.

3.4.4 Expected Return and Risk in the Risk Averse Category

In choosing the best portfolio seen from the risk-averse category, one practical aspect was the value of the expected return and the risk that would be accepted. Figures 9 and 10 describe each formed portfolio's expected return and risk values.

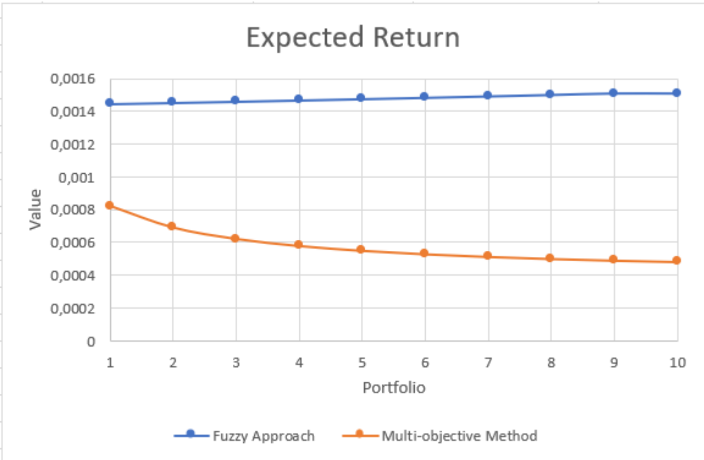

Figure 9: Expected Return with Fuzzy Approach and Multi-objective Method

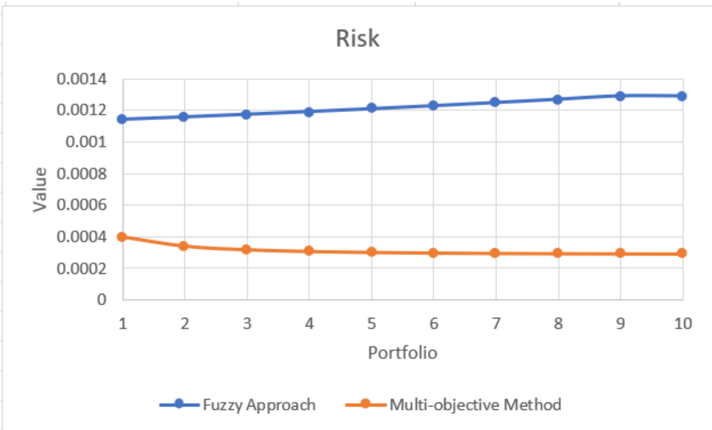

Figure 10: Risk with Fuzzy Approach and Multi-objective Method

Each portfolio's expected return and risk values had a corresponding line. In this case, the fuzzy approach had an expected return higher than the multi-objective method. The risk value of the portfolio also had the same tendency; namely, the fuzzy approach was much riskier than the multi-objective method for each structured portfolio.

Portfolio selection used two methods, namely the fuzzy approach and the multi-objective method, which had several characteristics of its own regarding the expected value return and the resulting risk. In this case, two categories of investors were distinguished: namely, risk-seeking ($f_1(x) \leq 0.001529$) and risk-averse ($f_1(x) \leq 0.0010443397$) for the fuzzy approach. The multi-objective method of risk-seeking classification was marked with a value $0.001 \leq k \leq 1$ and risk-averse with a value $2 \leq k \leq 10$.

The result was that the multi-objective method of expected return value in the risk-seeking category was above the expected return value generated by the fuzzy approach. Furthermore, it was commensurate with the resulting risk. However, in the risk-averse category, the expected return value with the fuzzy method was above the expected return value of the multi-objective method. It also corresponded to the resulting risk. Indeed, this thing became an additional preference of investors in the portfolio selection process in terms of methods and types of expected categories. This condition is because this method opens up opportunities to follow the extent to which investors want to get profits. These results also reinforce Subekti and Kusumawati (2015); Chen and Wei (2019); Mehlawat et al. (2020) findings, but the main difference in the findings of this study is the offer of a more diverse choice of portfolio schemes to suit investors' wishes. The following research gap is the development of fuzzy applications for portfolio preparation by considering multi-periods and accommodating short selling.

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