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By Al-Omari, Al-Nasser, Gogah

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Double acceptance sampling plan for time truncated life tests based on transmuted new Weibull-Pareto distribution

Amer I. Al-Omari*[a], Amjad D. Al-Nasser[b], and Fatima S. Gogah[c]

[a] *Department of Mathematics, Science Faculty, Al al-Bayt University, Mafraq, Jordan*
[b] *Quality Assurance and Institutional Effectiveness Center, Al Falah University, Dubai, UAE*
[c] *Department of Statistics, Science Faculty, Yarmouk University, Irbid, Jordan*

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In this paper, a two points acceptance sampling method were used to draw a decision on accepting or rejecting a tested product. It is also, assumed that the life time product follows a new distribution that formulated based on Weibull and Pareto life time distributions that it is known as new Weibull-Pareto (NWP) distribution. The mean life time in the second stage is obtained based on a pre-specified life time in the first one and the quality of a product is evaluated by computing the operating characteristic values and the minimum ratios of the mean life. The results are illustrated and a numerical example were given.

keywords: Double Acceptance Sampling Plan, Time Truncated Life Tests, New Weibull-Pareto Distribution, Operating Characteristic Function, Consumer's Risk.

1 Introduction

The cumulative distribution function of new Weibull-Pareto (NWP) random variable X is given by

$$F(x;\varphi,\eta,\psi) = 1 - \exp(-\varphi(\frac{x}{\eta})^{\psi}), x > 0, \eta > 0, \psi > 0, \varphi > 0, \quad (1)$$

*Corresponding author: alomari_amer@yahoo.com

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and the probability density function is

$$f(x;\varphi,\eta,\psi)=\frac{\varphi\psi}{\eta}(\frac{x}{\eta})^{\psi-1}\exp(-\varphi(\frac{x}{\eta})^{\psi}). \tag{2}$$

The mean and the variance of the NWP are defined as

$$\mu_X=\eta\varphi^{-\frac{1}{\psi}}\Gamma(\frac{\psi+1}{\psi}) \tag{3}$$

and

$$\sigma_X^2=\eta^2\varphi^{-\frac{2}{\psi}}\Gamma(\frac{\psi+2}{\psi})-\eta^2\psi^{-\frac{2}{\psi}}[\Gamma(\frac{\psi+1}{\psi})]^2 \tag{4}$$

The hazard rate function $H(x)$ of the NWP random variable is defined by

$$H(x)=\frac{\psi\varphi}{\eta^{\psi}}x^{\psi-1}. \tag{5}$$

For more details about the new Weibull-Pareto distribution see Nasiru and Luguterah (2015).

In this article ,a double acceptance sampling plan (DASP) developed for the truncated life test when the lifetime of a product is following NWP distribution. The main idea of using the DASP is that the decision could not be drawn based on the first sample, and to close the loop; another sample should be taken, which can be done by resampling more items; consequently the decision criterion will be improved based on the prior information that obtained from the first sample. The benefit of using the DASP is that the producer's risk will be reduced; which is a very important factor in quality control. Several authors considered DASP in there research work under different life time distributions. Aslam and Jun (2010), and Khan and Islam (2012) considered the generalized log-logistic distribution; Rao (2016) used the Marshall-Olkin extended exponential distribution. Recently, Ramaswamy and Anburajan (2012) used the generalized exponential distribution; and Gui (2014) suggested of using the Maxwell distribution; however, Malathi and Muthulakshmi (2015) considered the Marshall-Olkin extended exponential distribution.

In this paper, the suggested double acceptance sampling plan is given in Section 2. The operating characteristic function and the corresponding tables are given in Section 3. The minimum ratios to the specified life are presented in Section 4. Conclusions are given in Section 5.

2 Double acceptance sampling plan

The DASP based on truncated life time can be described as follows:

1. Chose a first random sample of size n_1 and put them on test during time t_0. If there are c_1 or fewer failures, accept the lot. If c_2+1 failures are observed, stop the test and reject the lot; i.e., $c_1<c_2$.

2. If the number of failures by t_0 is between c_1+1 and c_2, then select a second sample of size n_2 and then test the drawn items during another time t_0 . If at most c_2 failures are observed from the two samples, i.e., n_1+n_2, accept the lot. Otherwise, reject the lot and terminate the test.

Therefore, the DASP consists of four parameters $(n_1; c_1; n_2; and \quad c_2)$. Thus, the probability of acceptance the sampling plans $(n_1, c_1, \frac{t}{\mu_0})$ and $(n_2, c_2, \frac{t}{\mu_0})$ denoted by $L(p_1)$ and $L(p_2)$ can be obtained with respect the assumption that the lot is large enough in order to use the binomial probability distribution as follows

$$L(p_1) = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n_1-i} \tag{6}$$

$$L(p_2) = \sum_{i=0}^{c_2=2} \binom{n_2}{i} p^i (1-p)^{n_2-i} \tag{7}$$

where $p = F(t; \mu) = F(\frac{t}{\mu_0}.\frac{\mu_0}{\mu})$ is given in (1).

The probability of acceptance a lot is

$$L(p) = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \left(\sum_{j=0}^{c_2-j} \binom{n_2}{j} p^j (1-p)^{n_2-j} \right) \tag{8}$$

If $c_1 = 0$ and $c_2 = 2$,then the probability of acceptance is the total of three different probabilities that can be defined as:

$P(A)$= P(no failure occurs in sample 1) + P(1 failure occurs in sample 1 and 0 or 1 failure occurs in sample 2) + P(2 failures occur in sample 1 and 0 or 1 failure occurs in sample 2).

The values of the probability of acceptance a lot for a DASP using new Weibull-Pareto distribution are obtained at P^* with values 0.75, 0.90, 0.95, 0.99 and $\frac{t}{\mu_0}$ with values 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 are given in Tables 1 and 2 based on single and double acceptance sampling plans, respectively. These selections of P^* and $\frac{t}{\mu_0}$ are computable with Balakrishnan et al. (2007), Kantam et al. (2001), Al-Nasser and Al-Omari (2013), and Baklizi et al. (2005).

3 Operating characteristic function

The operating characteristics is the best quality tool to evaluate the sampling plan, in this trend and for a fixed parameters values $\varphi = 2$ and $\psi = 2$ in NWP distribution, the first stage of the sampling plan for obtaining the minimum sample size $(n_1, c_1 = 0, \frac{t}{\mu_0})$ is given in Table 1; extended the result for the second stage of the sampling plan,

$(n_1, n_2, c_1 = 0, c_2 = 2, \frac{t}{\mu_0})$ the OC function values are summarized in Table 2.

The results in Table (1) and Table (2) indicated that the OC values increases to unity with large ratio values $\frac{\mu}{\mu_0}$; noting that the probability based on DASP is exceed the probability values based on a single sampling plan.

4 Minimum ratio of th true mean life to the specified mean life and producer's risk

The producer's risk is known as the probability of rejection of a good lot. i.e., $(\mu \geq \mu_0)$. For the suggested DASP using NWP distribution and for a given value of the producer's risk θ, we want to find the minimum quality level of $\frac{\mu}{\mu_0}$ that will assert the PR to be at most θ. Therefore, $\frac{\mu}{\mu_0}$ is the smallest positive number for which $p = F(t;\mu) = F(\frac{t}{\mu_0}.\frac{\mu_0}{\mu})$ satisfies the inequality

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \left(\sum_{j=0}^{c_2-i} \binom{n_2}{j} p^j (1-p)^{n_2-j} \right) \geq 1-\theta \tag{9}$$

For the proposed acceptance sampling plan $(n_1, n_2, c_1 = 0, c_2 = 2, \frac{t}{\mu_0})$ at a specified consumer's confidence level P^* , the smallest values of $\frac{\mu}{\mu_0}$ satisfying (9) are summarized in Table 3.

Assume that the experimenter wants to assert that true unknown average life is at least 1000 hours with confidence level 0.90. And, assume that the acceptance numbers for this case are $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1 = 8$ and $n_2 = 14$. Thus, the lot is accepted if within 628 hours no failure is detected in a sample of size 8. For single sampling plan, the probabilities of acceptance the lot are 0.538215, 0.856523, 0.933483, 0.962021, 0.975525, 0.982939 for $\frac{\mu}{\mu_0} = 4, 6, 8, 10, 12$, respectively. Based on the DASP for the same measurements the probability values of accepting the lot are 0.820657, 0.993612, 0.999342, 0.999876, 0.999967, 0.999989. Also, we can note that as the ratio $\frac{\mu}{\mu_0}$ increases using DASP the probability of acceptance increases.

The producer's risk with respect to time of experiment for DASP using NWP distribution for $c_1 = 0$ and $c_2 = 2$ are presented in Table 4 for $P^* = 0.75, 0.90, 0.95, 0.99$. For example, when $\frac{\mu}{\mu_0} = 10$ (the unknown average life is ten times of specified average life) producer's risk when the experiment's times are 1571 hours and 4712 hours will be 0.000062 and 0.004098 , respectively. It is noted that for increases quality level of the product the producer's risk decreases.

5 Conclusions

This paper aims to reduce the producer's risk by considering the DASP assuming that the lifetime distribution is NWP. The optimal sample sizes in the two stages were obtained and the proposed sampling plan were evaluated by using the operating characteristics values as a quality tool. The results indicated that the DASP robustify the single sampling plan in terms of the operating characteristic values when the lifetime of a product following the NWP distribution.

Table 1: Operating characteristic values of the sampling plan $(n_1, c_1 = 0, \frac{t}{\mu_0})$ for a given P^* under the NWP with $\varphi = 2$ and $\psi = 2$

			$\frac{t}{\mu_0}$					
P^*	$\frac{t}{\mu_0}$	n_1	2	4	6	8	10	12
0.75	0.628	5	0.678965	0.907741	0.957892	0.976091	0.984632	0.989303
0.75	0.942	2	0.705769	0.91657	0.962021	0.978456	0.986158	0.990367
0.75	1.257	2	0.537684	0.856312	0.93338	0.961962	0.975486	0.982912
0.75	1.571	1	0.615944	0.885901	0.94758	0.970167	0.980803	0.986629
0.75	2.356	1	0.336255	0.761495	0.885947	0.93415	0.957341	0.970179
0.75	3.141	1	0.144112	0.616134	0.806348	0.88597	0.92544	0.947612
0.75	3.927	1	0.048414	0.469075	0.714308	0.827581	0.885929	0.91933
0.75	4.712	1	0.012784	0.336255	0.616071	0.761495	0.839976	0.885947
0.9	0.628	8	0.538215	0.856523	0.933483	0.962021	0.975525	0.982939
0.9	0.942	4	0.49811	0.840101	0.925485	0.957377	0.972508	0.980827
0.9	1.257	2	0.537684	0.856312	0.93338	0.961962	0.975486	0.982912
0.9	1.571	2	0.379387	0.784821	0.897907	0.941223	0.961974	0.973437
0.9	2.356	1	0.336255	0.761495	0.885947	0.93415	0.957341	0.970179
0.9	3.141	1	0.144112	0.616134	0.806348	0.88597	0.92544	0.947612
0.9	3.927	1	0.048414	0.469075	0.714308	0.827581	0.885929	0.91933
0.9	4.712	1	0.012784	0.336255	0.616071	0.761495	0.839976	0.885947
0.95	0.628	10	0.460994	0.823993	0.917556	0.952754	0.9695	0.978719
0.95	0.942	5	0.418463	0.804293	0.907741	0.947008	0.965754	0.976091
0.95	1.257	3	0.394267	0.792406	0.901754	0.943489	0.963456	0.974478
0.95	1.571	2	0.379387	0.784821	0.897907	0.941223	0.961974	0.973437
0.95	2.356	1	0.336255	0.761495	0.885947	0.93415	0.957341	0.970179
0.95	3.141	1	0.144112	0.616134	0.806348	0.88597	0.92544	0.947612
0.95	3.927	1	0.048414	0.469075	0.714308	0.827581	0.885929	0.91933
0.95	4.712	1	0.012784	0.336255	0.616071	0.761495	0.839976	0.885947
0.99	0.628	15	0.312999	0.747972	0.87892	0.929975	0.954601	0.96825
0.99	0.942	7	0.295338	0.737191	0.873266	0.926606	0.952386	0.966689
0.99	1.257	4	0.289104	0.73327	0.871199	0.925371	0.951573	0.966116
0.99	1.571	3	0.233682	0.695274	0.850839	0.913144	0.943507	0.960421
0.99	2.356	2	0.113068	0.579875	0.784902	0.872637	0.916502	0.941248
0.99	3.141	1	0.144112	0.616134	0.806348	0.88597	0.92544	0.947612
0.99	3.927	1	0.048414	0.469075	0.714308	0.827581	0.885929	0.91933
0.99	4.712	1	0.012784	0.336255	0.616071	0.761495	0.839976	0.885947

Table 2: Operating characteristic values of the sampling plan $(n_1, n_2, c_1 = 0, c_2 = 2, \frac{t}{\mu_0})$ for a given P^* under NWP with $\varphi = 2$ and $\psi = 2$.

				$\frac{t}{\mu_0}$					
P^*	$\frac{t}{\mu_0}$	n_1	n_2	2	4	6	8	10	12
0.75	0.628	5	10	0.926711	0.997996	0.999804	0.999964	0.99999	0.999997
0.75	0.942	2	5	0.935885	0.998284	0.999833	0.999969	0.999992	0.999997
0.75	1.257	2	3	0.888262	0.996651	0.999667	0.999938	0.999983	0.999994
0.75	1.571	1	3	0.873569	0.995883	0.999583	0.999922	0.999979	0.999993
0.75	2.356	1	2	0.707583	0.986433	0.998516	0.999715	0.999922	0.999974
0.75	3.141	1	2	0.373025	0.943436	0.992738	0.998517	0.999586	0.999856
0.75	3.927	1	2	0.138323	0.850342	0.976682	0.994874	0.998516	0.999475
0.75	4.712	1	2	0.037865	0.707583	0.943408	0.986433	0.995902	0.998516
0.9	0.628	8	14	0.820657	0.993612	0.999342	0.999876	0.999967	0.999989
0.9	0.942	4	7	0.791309	0.992127	0.999179	0.999845	0.999958	0.999986
0.9	1.257	2	4	0.835889	0.994345	0.999421	0.999891	0.999971	0.99999
0.9	1.571	2	3	0.73122	0.988738	0.9988	0.999771	0.999938	0.999979
0.9	2.356	1	2	0.707583	0.986433	0.998516	0.999715	0.999922	0.999974
0.9	3.141	1	2	0.373025	0.943436	0.992738	0.998517	0.999586	0.999856
0.9	3.927	1	2	0.138323	0.850342	0.976682	0.994874	0.998516	0.999475
0.9	4.712	1	2	0.037865	0.707583	0.943408	0.986433	0.995902	0.998516
0.95	0.628	10	16	0.745974	0.989516	0.998887	0.999788	0.999943	0.999981
0.95	0.942	5	8	0.705336	0.98691	0.998588	0.99973	0.999927	0.999975
0.95	1.257	3	5	0.684544	0.985443	0.998417	0.999696	0.999918	0.999972
0.95	1.571	2	4	0.638557	0.981509	0.997942	0.999602	0.999891	0.999963
0.95	2.356	1	2	0.707583	0.986433	0.998516	0.999715	0.999922	0.999974
0.95	3.141	1	2	0.373025	0.943436	0.992738	0.998517	0.999586	0.999856
0.95	3.927	1	2	0.138323	0.850342	0.976682	0.994874	0.998516	0.999475
0.95	4.712	1	2	0.037865	0.707583	0.943408	0.986433	0.995902	0.998516
0.99	0.628	15	22	0.542465	0.972437	0.996823	0.999377	0.999829	0.999941
0.99	0.942	7	11	0.507473	0.968158	0.99627	0.999264	0.999798	0.999931
0.99	1.257	4	6	0.532886	0.971637	0.996725	0.999358	0.999824	0.99994
0.99	1.571	3	5	0.417415	0.955141	0.994523	0.998903	0.999696	0.999895
0.99	2.356	2	3	0.247287	0.916102	0.988751	0.997668	0.999344	0.999772
0.99	3.141	1	2	0.373025	0.943436	0.992738	0.998517	0.999586	0.999856
0.99	3.927	1	2	0.138323	0.850342	0.976682	0.994874	0.998516	0.999475
0.99	4.712	1	2	0.037865	0.707583	0.943408	0.986433	0.995902	0.998516

Table 3: Minimum ratio of $\frac{t}{\mu_0}$ for the acceptability of a lot with producer's risk of 0.05 under NWP with $\varphi = 2$ and $\psi = 2$.

	$\frac{t}{\mu_0}$							
P^*	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	2.172	2.11	2.386	2.465	3.081	4.107	5.135	6.161
0.9	2.68	2.787	2.621	2.982	3.081	4.107	5.135	6.161
0.95	2.94	3.067	3.13	3.275	3.081	4.107	5.135	6.161
0.99	3.544	3.647	3.564	3.911	4.471	4.107	5.135	6.161

Table 4: Producer's risk with respect to time of experiment for double acceptance sampling based on NWP with $\varphi = 2$ and $\psi = 2$ for and $c_1 = 0$ and $c_2 = 2$

				$\frac{t}{\mu_0}$					
P^*	$\frac{t}{\mu_0}$	n_1	n_2	2	4	6	8	10	12
0.75	0.628	5	10	0.073289	0.002004	0.000196	0.000036	0.00001	0.000003
0.75	0.942	2	5	0.064115	0.001716	0.000167	0.000031	0.000008	0.000003
0.75	1.257	2	3	0.111738	0.003349	0.000333	0.000062	0.000017	0.000006
0.75	1.571	1	3	0.126431	0.004117	0.000417	0.000078	0.000021	0.000007
0.75	2.356	1	2	0.292417	0.013567	0.001484	0.000286	0.000078	0.000027
0.75	3.141	1	2	0.626975	0.056564	0.007262	0.001483	0.000414	0.000144
0.75	3.927	1	2	0.861677	0.149658	0.023318	0.005126	0.001484	0.000525
0.75	4.712	1	2	0.962135	0.292417	0.056592	0.013567	0.004098	0.001484
0.9	0.628	8	14	0.179343	0.006388	0.000658	0.000124	0.000033	0.000011
0.9	0.942	4	7	0.208691	0.007873	0.000821	0.000155	0.000042	0.000014
0.9	1.257	2	4	0.164111	0.005655	0.000579	0.000109	0.000029	0.00001
0.9	1.571	2	3	0.268781	0.011262	0.0012	0.000229	0.000062	0.000021
0.9	2.356	1	2	0.292417	0.013567	0.001484	0.000286	0.000078	0.000027
0.9	3.141	1	2	0.626975	0.056564	0.007262	0.001483	0.000414	0.000144
0.9	3.927	1	2	0.861677	0.149658	0.023318	0.005126	0.001484	0.000525
0.9	4.712	1	2	0.962135	0.292417	0.056592	0.013567	0.004098	0.001484
0.95	0.628	10	16	0.254026	0.010484	0.001113	0.000212	0.000057	0.000019
0.95	0.942	5	8	0.294664	0.01309	0.001412	0.00027	0.000073	0.000025
0.95	1.257	3	5	0.315456	0.014557	0.001583	0.000304	0.000082	0.000028
0.95	1.571	2	4	0.361443	0.018491	0.002058	0.000398	0.000109	0.000037
0.95	2.356	1	2	0.292417	0.013567	0.001484	0.000286	0.000078	0.000027
0.95	3.141	1	2	0.626975	0.056564	0.007262	0.001483	0.000414	0.000144
0.95	3.927	1	2	0.861677	0.149658	0.023318	0.005126	0.001484	0.000525
0.95	4.712	1	2	0.962135	0.292417	0.056592	0.013567	0.004098	0.001484
0.99	0.628	15	22	0.457535	0.027564	0.003177	0.000623	0.000171	0.000059
0.99	0.942	7	11	0.492527	0.031842	0.00373	0.000736	0.000202	0.000069
0.99	1.257	4	6	0.467114	0.028363	0.003275	0.000642	0.000176	0.000061
0.99	1.571	3	5	0.582585	0.04486	0.005477	0.001097	0.000304	0.000105
0.99	2.356	2	3	0.752713	0.083898	0.011249	0.002332	0.000656	0.000228
0.99	3.141	1	2	0.626975	0.056564	0.007262	0.001483	0.000414	0.000144
0.99	3.927	1	2	0.861677	0.149658	0.023318	0.005126	0.001484	0.000525
0.99	4.712	1	2	0.962135	0.292417	0.056592	0.013567	0.004098	0.001484

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