Preprocessing Techniques for the Multicast Problem in Wireless Networks

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Abstract: *Many features of the wireless ad hoc networks (WAHNs) like resource limitation, multi-hop communication, dynamic topology and lack of infrastructure make them very attractive and rise many very challenging optimization problems. In this paper we consider some preprocessing techniques usually used to reduce the size and, consequently, the solution time for several combinatorial optimization problems in telecommunication networks. In particular, for the Minimum Power Multicasting problem (MPM) in WAHNs, we describe the related network and mathematical models and subsequently propose some original reduction procedures, based on the graph model representation.*

Keywords: preprocessing techniques, multicast, wireless ad hoc networks

1. Introduction

Wireless networks are a special kind of networks in which the traditional cable links (of the wired networks) are replaced by radio links, an optimal way of communication between devices. There are infrastructured wireless networks and ad hoc wireless networks.

Wireless ad hoc networking is truly one of the most challenging and evolving research areas in both theory and practice but still more thorough theoretical investigation is required.

A *wireless ad hoc network* (*WAHN*) is an autonomous system consisting of mobile hosts connected by a shared wireless channel. Thus, the devices of the WAHN (called also *nodes*), randomly distributed in a given area, communicate with each other through a radio channel (called also *link*), without the use of any fixed infrastructure or centralized administration.

The major advantages of the WAHNs, like rapid deployment, robustness, independence of infrastructure, flexibility and support for mobility, made them very attractive and useful in several civil or military fields. The wide range of their applications includes emergency search-and-rescue operations, data acquisition operations in areas where natural disasters have destroyed the existing infrastructure, decision making in battlefield environment, video-conferencing, Internet access or exchanging information in buildings or trains, etc…

Unlike in wired networks, where a transmission from i to j generally reaches only node j, in wireless networks with omnidirectional antennas it is possible to reach several nodes with a single transmission, i.e. all the nodes within the transmission range of the communication from i to j (single-hop communication). This is the *wireless multicast advantage* (*WMA*) *property* ([5]).

Moreover, a terminal communicates indirectly with the others located out of its transmission range in a multi-hop fashion, using intermediate devices (acting as routers) that relay the data packets from the source of the messages to the destination nodes. In other words, a terminal is not only responsible for sending and receiving its own data, but it also has to forward the traffic of the others terminals.

Energy consumption is a very important issue in wireless networks since devices are usually powered by batteries. Therefore, by keeping transmission ranges small, the energy spent is reduced and the interference over the network is limited. In this way, the quality of the communication is improved.

This transmission mechanism and the WMA property are of great interest because their application permits to solve one of the most important problems of the WAHNs: the necessity of energy conservation during the communication.

2. Multicast communications model

The Multicast problem consists in connecting a specified node (called *source*) with a set of nodes (called *destinations*) with the possibility of using other devices of the network as relay nodes (*routers*).

We consider here the static WAHNs, that is the mobile terminals of the network are supposed to be stationary at the moment of the transmission. Furthermore, since the nodes are battery operated, power is likely to be a scarce resource. However, we assume that no power expenditure is involved in signal reception/processing activities. The interference problem is also omitted.

For a given network topology with a specified source node, the *Minimum Power Multicasting (MPM) problem* consists in assigning transmission powers to the devices of the network such that the connection between the source and all the destinations is guaranteed and the total energy consumption is minimized.

The MPM problem is a generalization of the well known Minimum Power Broadcasting (MPB) problem, the last one being obtained when the set of the destinations coincides with all of the nodes of the network (excepting the source). Moreover, since the MPM problem is NP-complete (so difficult to solve to optimality) and the MPB problem has been widely studied, most of the MPM formulations available today are an adaptation of the MPB models to the multicasting case.

Despite its great applicative importance, the MPM problem has not been studied enough but, because it is more general than the broadcasting version, more attention should be given to it.

Let $\mathrm{G}(\mathrm{V},\mathrm{A})$ be a directed complete graph, where the set of nodes V represents the set of devices of the network and the set of directed arcs A represents the transmission links between the nodes, i.e. all the possible pairs (i,j), with $i,j \in V$, $i \neq j$. Let $s \in V$ be the source of the messages and $D \subset V$ the set of the destination nodes. The other nodes of the network in the set $V \setminus (D \cup \{s\})$ may be involved in forwarding the packets (router nodes) or may neither receive nor transmit any message (isolated nodes).

By the assumption of static network, the distances d_{ij} between any two nodes i and j can be easily computed (therefore they are considered a priori known). Every arc $(i,j) \in A$ is associated with a cost p_{ij} representing the minimum quantity of power necessary to establish a direct communication from node i to node j. According to the commonly used, simple signal propagation model of Rappaport ([4]), the transmission power p_{ij} is proportional to a power of the distance d_{ij}, i.e. $p_{ij} = (d_{ij})^k$ where k (typically in the range between 2 and 5) is an environment-dependent exponent.

3. Preprocessing Techniques

The telecommunication networks problems (in particular, the MPM problem in WAHNs) can be usually formulated in terms of mixed, integer or $0-1$ linear programs (LP). Therefore, several logical processing procedures known in literature ([2]) can be applied to these problems.

In general, the preprocessing techniques consist in manipulating a formulation of a given mixed integer program (MIP) in order to decrease the overall time required for its solution. These procedures reduce the feasible region such that at least one of the optimal solutions remains feasible. Normally, the program solvers use different preprocessing techniques before starting to solve the LP relaxation by means of the Branch and Bound algorithm.

The preprocessing significantly modifies the given problem's formulation, by eliminating and substituting variables and constraints of the formulation and even by changing the coefficients of the constraints and those of the objective function. The solving programs solve this reduced formulation faster than the original one and successively find the values of the original variables.

The commonly used logical preprocessing techniques ([2]) include: inconsistent constraints identification, redundant constraints elimination and variables' bounds strengthening, coefficients improvement, probing, etc…

Moreover, the particular telecommunication networks problems' formulations considered here can be transformed in Set Covering formulations, allowing us to use for preprocessing not only the general techniques valid for all the integer or $(0-1)$ LP problems, but even those specific to the Set Covering problems: dominated rows and columns elimination rules, Lagrangian cost based fixing, lifting/projection methods, etc…

Since the MPM problem admits a Set Covering formulation ([3]), it is possible to apply to it all the above techniques.

Furthermore, besides these, we introduce in the sequel several original procedures which permit to reduce the size of an instance of the MPM problem, based on its graph model representation (described in the previous section).

These techniques are applicable also in the general case in which there is no symmetry between the transmission powers (i.e. $p_{ij} \neq p_{ji}, \forall i, j \in V, i \neq j$) or they do not simply depend on Euclidean distance.

The main goal of these preprocessing techniques is ***the elimination of the arcs (and possibly, the nodes) from the MPM problem's underlying graph.*** Thus, the various heuristics used for its solution, enriched with these reduction procedures, will produce an improved overall execution time.

A first arc elimination procedure is based on the observation (arising from the MPM problem definition) that the source transmits the message to the destinations, hence it does not have to receive the signal back. We have that:

All of the arcs entering the source node, that is having the form (i,s) for any $i \in V$, will be eliminated from the set A (exactly $|\delta_-(s)|$ arcs, that is as many as the input degree of nodes s).

Given an heuristic h for the solution of the MPM problem (for instance, one of the three multicasting versions of the *Broadcast Incremental Power* (*BIP*) algorithm [5]), let $c(h)$ be its cost.

If there is a transmission power p_{ij} such that $p_{ij} \geq c(h)$ then the corresponding arc (i, j) will be eliminated.

Consider now a non-destination node j and the minimal power required for reaching this node by any other node of the network. If this power exceeds the maximal power required for a direct transmission from the source to all the destinations, then the node j can be considered an isolated node (there is no reason for it to act as a router). This follows from the definition of the MPM problem, that involves the minimization of the total power necessary to make the message transmitted by the source reach all the destinations. Hence:

Let $j \notin D$. If the following inequality holds:

$$\min_{k \in V} p_{kj} > \max_{i \in D} p_{si}$$

then eliminate the node j from the set V and its incident arcs from the set A.

Moreover, if the power required to reach a non-destination node from a node $k \in V$ exceeds the maximal power necessary for a direct transmission from k to all of the destination nodes of the network then, for the same arguments as before, the arc (k, j) can be eliminated.

For all $k \in V, j \notin D$, if the following inequality is satisfied:

$$p_{kj} > \max_{i \in D} p_{ki}$$

then the arc (k, j) ***can be eliminated from the set*** A***.***

Now we propose a technique based on a property similar to the well-known *triangle inequality*, that states that the sum of the lengths of any two sides of a triangle always exceeds the length of the third side.

Considering the transmission powers associated with the arcs of the network's underlying graph for the MPM problem as sides' lengths, we observe that the above inequality does not generally hold.

In the general case (the powers are not exponentially depending on the distance), if the transmission powers do not verify one triangle inequality then, by the minimization condition, we can eliminate one arc from the graph. In other words, if the direct transmission from a node i to a node j is more "expensive" (in terms of power consumption) than the indirect one (using an intermediate node k as a router), then the route (i,j) has no use. Formally we have:

If for each $(i, j) \in A$***, exists*** $k \in V$***, different from*** i ***and*** j***, such that the following triangular-type inequality holds*** $p_{ik} + p_{kj} \leq p_{ij}$

then the arc (i, j) ***is a candidate for elimination from the set A.***

The arc's elimination will be effectively done after verifying the above inequality for all the triangles having (i, j) ***as a side. Moreover, if the verification is positive, the power*** p_{ij} ***can be updated to the value*** $p_{ik} + p_{kj}$***.***

Now, let k be a non-destination node, different from the source of the message. Assuming that we "pay" less power for a direct transmission from node i to node j than for the indirect one (routing by the node k), the node k will not be used as a router for transmitting the message from i to j. Therefore we have:

Let $k \notin D, k \neq s$ ***and*** $j \in V, j \neq k$***. If the triangular-type inequality***

$$p_{ik} + p_{kj} > p_{ij}$$

is satisfied for every $i \in V, i \neq j$***, then eliminate the arc*** (k, j) ***from the set A.***

Obviously, all the above preprocessing techniques can be iteratively applied in order to get all the possible nodes/arcs eliminations in the graph related to the MPM problem. With an adequate implementation (our future goal), they could be useful to obtain a drastic and rapid reduction of the instance size and, consequently, an improved time for its solution.

Moreover, the proposed procedures could be applied to all the telecommunications problems having a network model like that described in the previous section for the MPM problem.

In conclusion, our goal was the preprocessing of the graph in order to obtain a corresponding adjacency matrix of smaller dimension, that can be used as input data in the implementation of the Set Covering formulation already given for the MPM problem ([3]). Moreover, after all the possible nodes/arcs eliminations using the techniques introduced in this section, we note that to the MPM problem's underlying graph can be also applied the reduction procedures usually valid for the Steiner trees in wired networks ([1]). This is explained by the fact that the WMA is usually included in the problem's formulation and it is not necessarily involved in its network graph based model.

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